Subcarrier and Power Allocation Algorithm for Spectral Efficiency Maximization in Superposition Coding OFDMA Systems

Mateus de Paula Marques*
Taufik Abrão†

Electrical Engineering Department, State University of Londrina, Rod. Celso Garcia Cid - PR445, Londrina, PR, 86057-970, Po.Box 10.011, Brazil
†taufik@uel.br

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This paper addresses the optimization problem on subcarrier and power allocation of OFDMA system under spectral efficiency (SE) metric when deploying superposition coding transmission strategy. An algorithm with polynomial time complexity, of the order of $(UN\log_2(N))$ has been proposed for sub-optimal SE maximization. Results indicate that the system SE increases with the use of superposition coding technique. Besides, the throughput gain with superposition coding adoption increases when the number of users ($U$) approaches the number of subcarriers ($N$) available in the system.

Keywords: Resource allocation; spectral efficiency; energy efficiency; superposition coding; OFDMA.

1. Introduction

The popularization of mobile devices over the last years led to an exponential growth in data rate demand. While the telecommunications companies try to quickly expand their infrastructure to support the new scenario, many research groups over the world search for new solutions to improve the mobile networks performance aiming to surpass the current scarcity in high data rate connectivity.

Wireless cooperative networks are among many technologies that emerged in order to fulfill these new companies and users requirements. These networks are often more energy efficient than non-cooperative systems, since transmission power usually is not linearly proportional to the distance between transmitter and receiver. In cooperative systems each mobile terminal (MT) communicates with its respective...
base station (BS) through a relay station (RS) that may or may not be fixed. Besides energy efficiency (EE), other characteristics may be improved when cooperatives scenarios are considered, such as spectral efficiency (SE), system throughput and average transmission power (ATP).

In order to further improve the network one may use direct sequence code division multiple access combined with multi-carrier techniques like the orthogonal frequency division multiplexing, namely multi-carrier direct sequence code division multiple access (MC-DS/CDMA) networks, which are characterized by the division of the total available spectrum into parallel uncorrelated non-selective CDMA sub-channels, which improves granularity and it is responsible for the enhancement of the system EE and SE as well as lowering ATP.

Since users may connect to the network to use different multimedia services such as voice calls, video calls, video streaming, data transfer and so forth, the system must support a multi-rate scenario. Orthogonal frequency division multiple access (OFDMA) systems may implement adaptive modulation and coding techniques in order to create such flexibility and support from voice service to high data rate services.

One of possible PHY techniques aiming to improve spectral efficiency includes superposition coding (SC). Superposition coding for broadcast channels was suggested by Cover in 1972 [1]. Suppose two messages destined to two users are queued up for transmission at a base station. Instead of splitting transmit resources as space or time or frequency division multiplexing, a superposition encoder jointly encodes both the messages under the same total transmit power constraint as in multiplexing schemes. Therefore, both messages can be sent simultaneously while share the transmitter resources.

Indeed, message symbols of one user will appear as interference to the other, both the receivers can decode their messages by deploying successive interference cancellation (SIC) technique. Furthermore, for broadcast channels, the SC technique is able to improve aggregate user throughput compared to an orthogonal multiplexing scheme.

Theoretically, SC scheme provides a suitable bound of extended capacity in the broadcast channels. However, such performance has not comprehensively evaluated in the experimental and practical domain. The reason is that such performance gain is under the strict assumptions, including perfect synchronization and error-free feedback, which may not be practical in several practical cases. Physical layer implementation of an OFDM-based superposition coding system in software defined radio, including discussion of the functional blocks, packet structure, and receiver algorithms has been presented in [2, 3].

The combination of different techniques may be a start solution to the next generation networks which aim to be green, profitable and provide excellent user experience. For instance, a different approach to improve the packet delivery efficiency on a vulnerable downlink using superposition coding and a multiuser transmission
scheme that deliberately introduces interference among signals at the transmitter has been proposed in [4]. Authors show that a transmitter serving multiple links can use simple two-user superposition codes to improve the packet delivery efficiency on its most vulnerable links. Indeed, superposing signals of far-away users on to those of high-traffic users yields maximum benefit, which implies that the degrees-of-freedom gain in doing so compensates the increased interference from superposition signals.

In this work, an optimization procedure based on the classical water-filling algorithm (WFA) is proposed to solve the Joint Subcarrier and Power Allocation (JSPA) problem for SE maximization in a SC-OFDMA system. The main contribution of this work is the proposition of an algorithm with polynomial time complexity, of the order of $\mathcal{O}(UN \log_2(N))$. The proposed JSPA algorithm organizes subcarriers through a binary search tree (BST) procedure in order to reduce the search time to $\mathcal{O}(\log_2(N))$. Furthermore, with the adoption of the proposed procedure we have confirmed the gain due to the application of SC increases when the number of users approaches the number of subcarriers.

The rest of this paper is organized as follows. In Section 2 the system model for the SC-OFDMA network is described, while in Section 3 the SE optimization problem is depicted along with the JSPA-SC algorithm. Afterward, in Section 4 the numerical results are discussed assuming realistic networks operation scenarios. Finally, the main conclusions are offered in Section 5.

2. System Model

In the downlink of a superposition coding orthogonal frequency division multiple access (SC-OFDMA) system, the transmission for $U$ users is considered along $N$ subcarriers, which can be shared among users due to the the superposition coding (SC) technique. Let $\zeta_{i,n}$ be the binary variable that tells if the $n$-th subcarrier is allocated for user $i$. Hence, if subcarrier $n$ is allocated for user $i$, $\zeta_{i,n} = 1$; otherwise, $\zeta_{i,n} = 0$.

Notice that the maximum number of users able to share a subcarrier depends on the diversity gain associated to the channel, while multiple access interference (MAI) on each subcarrier deteriorates the performance, as well as the associated spectral efficiency gain. Furthermore, the error propagation due to the deployment of SIC detectors at the SC-OFDMA receivers makes the channel sharing unfeasible beyond some units of users per subcarrier. On the other hand, a substantial gain in terms of SE is achieved for two users sharing a subcarrier through SC transmitting scheme.

Hence, for the sake of simplicity and taking into account the tradeoff between the aforementioned SE gain and the increasing MAI with the number of users, in this work we have considered a maximum of two users sharing the same subcarrier; although this limit can be extended if the transmission is feasible w.r.t. the BER and transmission power levels, at the cost of higher complexity. Hence, this condition is
expressed by:

$$\sum_{i=1}^{U} \zeta_{i,n} \leq 2, \quad \forall n \in \mathcal{N}$$  \hspace{1cm} (1)

where $\mathcal{N}$ is the set of all subcarriers available in the system. We have assumed perfect channel state information (CSI) at the transmitter side, i.e., there is no error in the channel estimation process.

The bit error rate (BER) is a key metric for quality of service (QoS), and it is related to the signal-to-noise-plus-interference ratio (SNIR) as follows:

$$\gamma_{i,n} = \frac{p_{i,n}|g_{i,n}|^2}{\sum_{j \in \mathcal{I}_{i,n}} p_{j,n}|g_{i,n}|^2 + \sigma^2}$$  \hspace{1cm} (2)

where $|g_{i,n}|$ is the amplitude channel gain of the $i$-th user on the $n$-th subcarrier, $p_{i,n}$ is the transmission power, $\sigma^2$ is the additive white gaussian noise (AWGN) variance at the receiver’s input, and $\mathcal{I}_{i,n}$ is the set of users interfering on the signal of user $i$ in subcarrier $n$. In this work, since the limit of two users per subcarrier is considered, we have $||\mathcal{I}_{i,n}|| \leq 1, \forall i \in \mathcal{U}$, and $\forall n \in \mathcal{N}$; the set of all users is defined by $\mathcal{U}$ and $||\cdot||$ is the cardinality of the set.

Let $w$ be the bandwidth of the subcarriers. The total achievable rate of user $i$ is given by:

$$r_i = w \sum_{n=1}^{N} \zeta_{i,n} \log_2(1 + \theta_{i,n}^{\text{max}} \gamma_{i,n})$$  \hspace{1cm} (3)

where $\theta_{i,n}^{\text{max}}$ is the inverse of the gap between the theoretical bound and the real information rate. Let $\text{BER}_{i}^{\text{max}}$ be the maximum tolerable BER for user $i$. Usually, the gap $\theta_{i,n}^{\text{max}}$ can be approximated by [5]:

$$\theta_{i,n}^{\text{max}} = -\frac{1.5}{\ln(5\text{BER}_{i}^{\text{max}})}$$  \hspace{1cm} (4)

This way, the minimum SNIR for user $i$ on subcarrier $n$ is given by:

$$\gamma_{i,n}^{\text{min}} = -\frac{2}{3} \ln(5\text{BER}_{i}^{\text{max}})(2^{S_{i,n}} - 1)$$  \hspace{1cm} (5)

where $S_{i,n}$ is the spectral efficiency (SE) achieved by user $i$ on subcarrier $n$. It means that if the $i$-th user achieves a SE on subcarrier $n$, meaning that $\zeta_{i,n} = 1$ and $p_{i,n} > 0$, one can obtain the minimum SNIR in which its possible to achieve $S_{i,n}$ with $\text{BER}_{i} \leq \text{BER}_{i}^{\text{max}}$.

In this work, the superposition coding technique is used in order to increase the system capacity, considering at most two users transmitting at the same subcarrier. It is known that SC takes advantage of the near far (NF) effect in order to transmit for more than one user in the same frequency band [1, 6]. Hence, considering the transmission for two users in the same subcarrier $n$, one user located in the border
Subcarrier and Power Allocation in Superposition Coding OFDMA Systems

5

of the cell, namely degraded user, \(d\), while the other one is placed close to the base station (BS), namely potential user, \(p\); hence, the receiving processes is given as follows. The far user \(d\) detects its signal deploying a single user detector (SUD), treating the signal of potential user \(p\) as noise. On the other hand, the potential user who has a better channel condition, performs a successive interference cancellation (SIC), obtaining its signal free from interference, hence:

\[
|I_{d,n}| = 1, \quad |I_{p,n}| = 0
\]  

(6)

Moreover, in order to keep the transmission of the potential user \(p\) feasible on the \(n\)th subcarrier, its channel gain must be considerably greater than the corresponding channel gain of the degraded user:

\[
\frac{|g_{p,n}|^2}{|g_{d,n}|^2} = \alpha \gg 1
\]  

(7)

If this relation holds, we can establish the maximum tolerable BER for the degraded \(d\)-user to be detected by the potential \(p\)-user receiver, so that the SIC is possible with small probability of error propagation. We recall that the number of users sharing the same subcarrier can be increased if there is enough diversity among the channels. For simplicity, in this work only one class of service is considered, which is a real-time service that has relatively low requirements on BER\(_{\text{max}}\). Thus, if \(\text{BER}_{i}^{\text{max}} = 10^{-3}, \forall i \in \mathcal{U}\), then the maximum tolerable BER for the detection of the degraded \(d\)-user by the potential \(p\)-user is assumed to be \(\text{BER}_{dp}^{\text{max}} = 10^{-9}\). This way, it is reasonable that the user \(p\) is able to detect its signal under a \(\text{BER}_{p}^{\text{max}} = 10^{-3}\).

However, if these conditions do not hold, the SC is not adopted for the subcarrier \(n\). From this, we can establish:

\[
\gamma_{d,n}^{\text{min}} = -\frac{2}{3} \ln(5 \cdot 10^{-3})(2^S_{d,n} - 1)
\]  

(8)

\[
\gamma_{dp,n}^{\text{min}} = -\frac{2}{3} \ln(5 \cdot 10^{-9})(2^S_{d,n} - 1)
\]  

(9)

where \(\gamma_{d,n}^{\text{min}}\) and \(\gamma_{dp,n}^{\text{min}}\) are the minimum SNIR’s for the signal of user \(d\) detected on himself and on \(p\), respectively. As a consequence, the \(\alpha\) constant can be readily found:

\[
\alpha = \frac{\gamma_{dp,n}^{\text{min}}}{\gamma_{d,n}^{\text{min}}} = \frac{\ln(5 \cdot 10^{-9})}{\ln(5 \cdot 10^{-3})} = 3.6075
\]  

(10)

Hence, the channel gain of the potential user \(p\) must be \(|g_{p,n}|^2 > 3.6075 \cdot |g_{d,n}|^2\) in order to adopt the linear superposition of signals. Note that this condition does not depend on the SE achieved by user \(d\) in \(n\). Thus, if there is any, this relation must hold.

3. Spectral Efficiency Maximization in SC-OFDMA

The well known SE maximization problem which can be solved by the water-filling algorithm [7] is considered in this work. Thus, the spectrally-efficient SC-
OFDMA design \((\eta_S)\) can be formulated as:

\[
\text{maximize } \eta_S = \sum_{i=1}^{U} \sum_{n=1}^{N} \zeta_{i,n} \log_2 \left(1 + \theta_i^{BER} \gamma_{i,n}\right) \tag{11a}
\]

subject to

\[
\begin{align*}
\text{c.1} : & \quad \sum_{n=1}^{N} p_{i,n} \leq P_{i,\text{max}}, \quad \forall i \in U \\
\text{c.2} : & \quad p_{i,n} \geq 0, \quad \forall i \in U, \forall n \in N \\
\text{c.3} : & \quad \sum_{i=1}^{U} \zeta_{i,n} \leq 2, \quad \forall n \in N 
\end{align*}
\tag{11b-11d}
\]

where Eqs. (11b), (11c) and (11d) are the constraints of maximum transmission power per user, non-negative power allocation and maximum number of users per subcarrier, respectively; note that \(P_{i,\text{max}}\) is the maximum transmission power allowed for user \(i\) and \(U_i(r_i)\) is the utility function of user \(i\), which is assumed to be a function of \(r_i\); indeed, the utility function can assume value 1 for SE maximization, or \(\log_2(r_i)\) for SE maximization with proportional fairness. In this work, we have assumed:

\[
U_i(r_i) = \log_2(r_i) \tag{12}
\]

Furthermore, there are other approaches in defining the utility function; for instance, a discussion on the rate control policies for communication networks can be found in [8]. The joint subcarrier and power algorithm is developed in two steps: firstly, the joint subcarrier and power allocation is performed without the SC technique; secondly, a refining process is carried out in order to allocate the subcarriers deploying the SC technique. The pseudo-code for the proposed joint subcarrier and power allocation is depicted in Algorithm 1. The steps of the algorithm are described in the sequel.

### 3.1. Joint Subcarrier and Power Allocation

From the spectral efficiency optimisation problem, eq. (11a)-(11d), it is clear that our problem is a mixed-integer programming problem, which generally requires exponential time to solve. Hence, in order to keep the problem in the continuous domain we relax the binary variables \(\zeta_{i,n}\) such that they are allowed to take any real values in the interval \([0,1]\).

**Lemma 1.** A necessary condition for \(\zeta_{i,n}\) being positive at the optimal solution is:

\[
i = \arg \max_{1 \leq j \leq U} \left\{ U_j'(r_j) \log_2(1 + \theta_j^{REM} \gamma_{j,n}) \right\} \tag{13}
\]

where \(U_j'(r_j)\) is any utility function of user \(i\), which is a smooth non-decreasing function of \(r_i\).
Algorithm 1 Joint Subcarrier and Power Allocation (JSPA) Algorithm

Initialization
01. generate $B_i, \forall i;$
02. $\mathcal{N}_i \leftarrow \emptyset, \forall i;$

First Step - JSPA
03. for $m = 0 : N - 1$
04. get the candidate subcarrier $g_{i,m}, \forall i \in \mathcal{U}$ from $B_i$
05. if $m = 0$
06. initialize the water-levels $l_{i,0}, \forall i \in \mathcal{U}$ with Eq. (19);
07. else
08. update the water-levels $l_{i,m}, \forall i \in \mathcal{U}$ with Eq. (17);
09. endif
10. update $r_i, \forall i$ with Eq. (3);
11. if there is any priority conflict
12. $\mathcal{N}_j \leftarrow g_{j,m} \cup \mathcal{N}_j$ with $j$ satisfying Eq. (13);
13. undo $l_{i,m}$ update for any conflicted user $k \neq j$;
14. undo $r_i$ update for any conflicted user $k \neq j$;
15. endif
16. $\mathcal{N}_i \leftarrow g_{i,m} \cup \mathcal{N}_i, \forall i \neq j, k \in \mathcal{U};$
17. endfor

Second Step - JSPA-SC
18. for $m = 0 : N - 1$
19. get $g_{i,m}, \forall i \in \mathcal{U}$ from $B_i$ satisfying Eq. (5);
20. update the water-levels $l_{i,m}, \forall i \in \mathcal{U}$ with Eq. (17);
21. update $r_i, \forall i$ with Eq. (3);
22. if there is any priority conflict
23. $\mathcal{N}_j \leftarrow g_{j,m} \cup \mathcal{N}_j$ with $j$ satisfying Eq. (13);
24. undo $l_{i,m}$ update for any conflicted user $k \neq j$;
25. undo $r_i$ update for any conflicted user $k \neq j$;
26. endif
27. $\mathcal{N}_i \leftarrow g_{i,m} \cup \mathcal{N}_i, \forall i \neq j, k \in \mathcal{U};$
28. endfor

End

$B_i$: BST of the $i$-th user.
$\mathcal{N}_i$: Set of the subcarriers allocated for user $i$.

Proof. Any optimal solution to the relaxed problem has to satisfy the following
Karush-Kuhn-Tucker (KKT) conditions:

\[
\begin{align*}
\mathcal{U}_i^\prime(r_i) \log_2(1 + \theta_i \gamma_{i,n}) - \lambda_n + \kappa_{i,n} &= 0 \quad (14a) \\
\frac{\zeta_{i,n} \theta_i \gamma_{i,n}}{\left(\tau_i p_j,n + \sigma^2_{g_i,n} + \theta_i \gamma_{i,n} p_i,n\right) \ln(2)} - \lambda_n + \kappa_{i,n} &= 0
\end{align*}
\]

\[
\sum_i \zeta_{i,n} \leq 2, \quad \forall n \quad (14b)
\]

\[
\sum_n p_{i,n} \leq P_{i,max}, \quad \forall i \quad (14c)
\]

\[
\zeta_{i,n} \mu_{i,n} = 0, \quad \forall i, n \quad (14d)
\]

\[
p_{i,n} \pi_{i,n} = 0, \quad \forall i, n \quad (14e)
\]

\[
p_{i,n}, \zeta_{i,n}, \kappa_{i,n}, \pi_{i,n} \geq 0, \quad \forall i, n \quad (14f)
\]

where \(\tau_i = |\mathcal{I}_{i,n}|\), \(\lambda_n\) and \(\kappa_{i,n}\) are the KKT multipliers for constraints (11d) and (11b), respectively, while \(\mu_{i,n}\) and \(\pi_{i,n}\) are the KKT multiplies for non-negativity of \(\zeta_{i,n}\) and \(p_{i,n}\), respectively. Hence, solving (14f), we get (complementary slackness condition):

\[
\begin{align*}
\zeta_{i,n} \left[\mathcal{U}_i^\prime(r_i) \log_2(1 + \theta_i \gamma_{i,n}) - \lambda_n + \kappa_{i,n}\right] &= 0 \quad (15a) \\
\zeta_{i,n} \left[\mathcal{U}_i^\prime(r_i) \log_2(1 + \theta_i \gamma_{i,n}) - \lambda_n\right] &= 0 \quad (15b)
\end{align*}
\]

since \(\zeta_{i,n} \geq 0\):

\[
\mathcal{U}_i^\prime(r_i) \log_2(1 + \theta_i \gamma_{i,n}) \geq \lambda_n \quad (16)
\]

if \(\zeta_{i,n} > 0\), equality holds, achieving an upper bound \(\lambda_n\). Thus, the proposition follows.

Although this proposition gives a necessary condition for the optimal values of \(\zeta_{i,n} > 0\), it cannot be used for solving the optimal solution since the objective function, in general, is neither a convex nor a concave function of \(\zeta_{i,n}\) and \(p_{i,n}\). On the other hand, it can be useful as a metric for guiding our subcarrier allocation strategy and algorithm. In this work, our algorithm allocates sorted subcarriers one-by-one. Thus, the \(n\)-th subcarrier will be allocated for the user that satisfies Eq. (13).

We recall that our algorithm allocates subcarriers in two steps; firstly, considering individual subcarrier allocation and secondly considering the linear superposition of signals. For the two steps, the choosing of the best user will be based in Eq. (13). Although, for the second step, the candidate users must satisfy one more condition given that its channel gain must be considerably greater than the one of the user already allocated on the subcarrier through step one. Thus, the candidate user must satisfy Eqs. (13) and (7).
Usually, low complexity joint subcarrier and power allocation algorithms require \( N \) iterations to allocate \( N \) subcarriers for users. In each iteration, water-filling algorithm is performed with time complexity \( O(N) \) in order to find the best users for each subcarrier (Lemma 1). Hence, the time complexity for each iteration is \( O(KN) \). Thus, the total complexity of these algorithms without superposition coding are often in the order of \( O(KN^2 \log_2(N)) \). In this work, we propose a joint subcarrier and power allocation algorithm for a SC-OFDMA system with time complexity \( O(KN \log_2(N)) \). This can be achieved by using a special data structure for organizing subcarriers called binary search tree (BST), along with the per-subcarrier water-level updating adopted, which keeps water-filling time complexity constant on each iteration. Hence, for the \((m_i + 1)\)-th subcarrier to be allocated for user \( i \), the water-level \( \ell_i \) is obtained by:

\[
\ell_i = \begin{cases} 
m_i \ell_i + h_{i,m_i+1}^{-1}, & \text{if } h_{i,m_i+1} < \ell_i \\
\ell_i, & \text{otherwise}
\end{cases}
\]

(17)

where \( m_i \) is the number of subcarriers already allocated for user \( i \), and \( h_{i,m} \) is given by:

\[
h_{i,m} = \frac{\theta_{\text{max}} |g_{i,m}|^2}{\sigma^2 + \sum_{j \in I_i} p_j |g_{i,m}|^2}
\]

(18)

For the first subcarrier allocated to user \( i \) (\( m_i = 0 \)):

\[
\ell_i = P_{i,\text{max}} + h_{i,1}^{-1}
\]

(19)

Thus, the proposed approach is to set the power level in the first subcarrier of each user to \( P_{i,\text{max}} \), and then gradually reduce the water-level according to the instantaneous channel gains condition of the new subcarriers. In fact, we are dealing with the classical water-filling solution where the objective is to find the root of the function:

\[
f(\ell_i) = \sum_{n=1}^{N} (\ell_i - h_{i,m}^{-1})^+ - P_{i,\text{max}}
\]

(20)

The proposed approach for dynamic water-level calculation in eq. (17) is identical to the classical water-filling algorithm if the subcarriers have already been allocated to the user. Although, in this work the water-level is dynamically calculated during the subcarrier allocation process.

### 3.1.1. Binary Search Tree Data Structure (BST)

In this work, the subcarriers are organized according to a binary search tree data structure (BST). The BST is a node-based data structure which allows at most two children per node [9]. A very important aspect of the BSTs is the balancing regarding the depth of the leaves. If the depth of the leaves differs at most by one, the BST is said to be balanced. A balanced BST has the nice property that
any leave is located in the distance of \(\lceil \log_2(N) \rceil \) from the root. Hence, important properties of balanced BSTs can be highlighted:

a) The time complexity to create a balanced BST is \(O(N \log_2(N))\);
b) The time complexity to print all the sorted elements in a balanced BST is \(O(N \log_2(N))\);
c) If the BST is balanced, any element of the BST can be reached with \(O(\log_2(N))\) operations.
d) Given a node \(n\), any node \(m\) and \(o\) located in the right and left sub-tree of \(n\), respectively, will satisfy: \(o < m\).

From our proposition, each user must have a BST of its subcarriers \((B_i)\). Moreover, the first part of the subcarrier allocation algorithm must fulfill the following steps: Firstly, the BST of each user must be created, so the time complexity is \(O(KN \log_2(N))\); Secondly, the BST of each user must be traversed in decreasing order, which is (computationally) the same as to print all the sorted elements of the BST, in order to test all the candidate subcarriers for the \(i\)-th user, obtaining the time complexity \(O(KN \log_2(N))\) again. Since water-filling complexity is constant on each candidate subcarrier test, Eq. (17), the total complexity of these steps is given by \(O(2KN \log_2(N)) \approx O(KN \log_2(N))\).

Each node of \(B_i\), \(\forall i\) must keep the following data:

- \(|g_{i,n}|^2\), \(\forall n \in \mathcal{N}\);
- \(\zeta_{i,n}\) and if \(\zeta_{i,n} = 0\), \(|g_{j,n}|^2\) must be kept, where \(\zeta_{j,n} = 1\);
- \(N_R\) and \(N_L\), as the number of unallocated subcarriers in the right and left sub-tree.
- \(N^o_R\) and \(N^o_L\), as the number of subcarriers allocated for other users in the right and left sub-tree.

Aiming to simplify our algorithm, we introduce the concept of priority of subcarriers for each user in order to guide the search for candidate subcarriers. The priority \(\psi_{i,n}\) of a subcarrier \(n\) to an user \(i\) is such that, if \(\psi_{i,n} < \psi_{i,m}\), then \(g_{i,n} > g_{i,m}\). Thus, the priority can be viewed as the position of the subcarrier in a vector sorted in decreasing order. The candidate subcarrier of each user will then be compared only with the subcarriers of the other users in the same priority level, which we call a priority conflict. It means that any subcarrier already allocated for any user will not be considered candidate. Hence, if a candidate subcarrier \(n\) for user \(i\) is not in the same priority level for the other users, this subcarrier is directly allocated for \(i\), without considering condition (13). This is fairly reasonable since this subcarrier has greater priority for user \(i\) than for the others.

Given the first step of subcarrier allocation, we turn now for the allocation of subcarriers using superposition coding. Let the node \(m\) of \(B_i\) be the one that contains the first subcarrier allocated for user \(i\) in which the water-filling solution assigned \(p_{i,m} = 0\). The search for candidate subcarriers to allocate using superposi-
tion coding for user $i$ will be done in the right sub-tree of node $n$. This is reasonable since water-filling solution will not assign any power level for any subcarrier $o$ with $g_{i,o} < g_{i,m}$. Note that the candidate subcarrier for any user $i$ on this step is the one which has been allocated for only one user $j$, with $j \neq i$, and that satisfies Eq. (7). The time complexity of this step is again $O(KN \log_2(N))$, so the total computational complexity for the joint subcarrier and power allocation algorithm is $O(KN \log_2(N))$.

4. Numerical Results

In this section, the numerical results obtained for the proposed algorithm are presented. The SC-OFDMA resource allocation simulations were carried out within the MatLab 7.0 platform. We assumed a rectangular cell of dimensions $x = y = 1$ Km, with one base station in the center and the users uniformly spread across all the cell extension. We considered a path loss exponent $\beta = 3.5$, and a log-normal medium-scale fading (shadowing) with normalized mean and variance equal to 7dB. Furthermore, we have assumed i.i.d. flat Rayleigh small-scale fading channels with mean 0 dB and variance 7dB. The adopted PSD of the background noise is $S_n = 174 \text{dBm} \text{Hz}$, and a total system bandwidth of $W = 10 \text{MHz}$.

In the following numerical results the total achieved rate in [Mbits/s] and the number of allocated subcarriers against iterations of the proposed algorithm have been characterized for the simple representative case of $U = 2$ users. Besides, the increasing of system throughput in [Mbits/s] against the number of users in the range $U \in [2; 30]$ has been analysed for the cases $N = [U; 2U; 10U]$. We have analysed the system throughput gain for both joint subcarrier and power allocation algorithms, i.e., with and without superposition coding, JSPA and JSPA-SC, respectively.

Figs. 1 and 2 depict the number of subcarriers allocated per user, along with the individual and total rates evolution as a function of the number of iterations for both JSPA algorithms, considering two cases: $N = U$ and $N = 2U$, respectively, both with a total number of users $U = 2$. Notice that in both cases, there is a substantial reduction in the information rate of the degraded user, due to the high power levels assigned by the potential user in the shared subcarriers. Hence, we need to establish a maximum power criterion for potential users in shared subcarriers. Furthermore, system throughput is considerably increased with the aid of superposition coding, mainly when the available number of subcarrier grows; hence one can conclude that adopting SC technique increases the system throughput.

In order to analyze the system throughput tendency deploying JSPA and JSPA-SC algorithms, Fig. 3 shows the overall throughput as a function of the number of users. It is clear that adopting superposition coding increases the system throughput in both cases analyzed, i.e., JSPA with and without SC. It can be seen that the gain in adopting SC w.r.t. the system throughput is much more remarkable when the system experiments the critical condition $U = N$. 
5. Conclusions

In this work, spectral efficiency maximization has been addressed in a superposition coding OFDMA system. An algorithm with $O(UN \log_2(N))$ time complexity for joint subcarrier and power allocation (JSPA) has been proposed in order to achieve quasi-optimal spectral efficiency maximization. The numerical results have been indicated that the overall spectral efficiency increases substantially with the use of superposition coding technique, specially when the number of users approaches the number of subcarriers available in the system ($N \rightarrow U$).

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Bibliography

Subcarrier and Power Allocation in Superposition Coding OFDMA Systems

Figure 2. Individual rates and number of subcarriers evolution for $U = 2$ and $N = 2U$. Single trial evaluation.

Figure 3. System throughput as a function of the number of users. 1000 trials.


