PSO Assisted Multiuser Detection for DS-CDMA Communication Systems

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Abstract

In this chapter, a heuristic perspective for the multiuser detection problem in the uplink of direct sequence code division multiple access (DS-CDMA) systems is discussed. In particular, the particle swarm optimization multiuser detector (PSO-MuD) is analyzed regarding several figures of merit, such as symbol error rate, near-far and channel error estimation robustness, and computational complexity aspects.

The PSO-MuD is extensively evaluated and characterized under different channel scenarios: additive white Gaussian noise (AWGN), single input single/multiple output (SISO/SIMO) flat Rayleigh, and frequency selective (multipath) Rayleigh channels.

Although literature presents single-objective (SOO) and multi-objective optimization (MOO) approaches to deal with multiuser detection problem, in this chapter the single-objective optimization criterion is extensively used, since its application requirement is simpler than the MOO, and its performance results for the proposed optimization problem are quite satisfactory. Nevertheless, the MOO is shortly addressed as an alternative approach.

Furthermore, the complexity × performance trade-off of the PSO-MuD is carefully analyzed via Monte-Carlo simulation (MCS), and the complexity reduction concerning the optimum multiuser detector (MuD) is quantified. Simulation results show that, after convergence, the performance reached by the PSO-MuD is much better than the conventional detector (CD), and somewhat close to the single user bound (SuB), having computational complexity substantially lower than OMuD.

Keywords: computational complexity; multiple access wireless communication systems; DS-CDMA; multiuser detection; particle swarm optimization; SIMO; (single-)multiple-objective optimization.
1 INTRODUCTION

The present and next generations of wireless communications demand high data transmission rates and good quality of service, so as to provide multimedia applications, such as video, audio, Internet access, among others. The big challenge in these systems can be summarized as: how can we improve efficiency, capacity and deal with spectrum scarcity? Combining different forms of diversity in multiple access systems is the key to deal with these limitations. In addition, different modulation formats can be efficiently and adaptively adopted, such as binary and quadrature phase shift keying (BPSK and QPSK, respectively), high order modulation ($M$-QAM), depending on the quality of the channel. Error correcting codes are also indispensable. Furthermore, multiuser detection (MuD) in multiple access systems, such as DS-CDMA, is another alternative to deal with these limitations.

In a DS-CDMA system, simple solutions like just a conventional detector may not provide a desirable quality of service, once the system capacity is strongly affected by multiple access interference (MAI). The capacity of a DS-CDMA system under practical scenarios is limited mainly by the MAI, self-interference (SI), near-far ratio (NFR) and fading channel effects. In order to reduce fading impact, the conventional receiver (Rake receiver) explores the path diversity, but it is not able to mitigate neither the MAI nor the NFR effects [1, 2]. In this context, multiuser detection emerged as a solution to overcome the MAI [2]. The best performance is acquired by the optimum multiuser detection (OMuD), based on the log-likelihood function (LLF) [2]. However, this is achieved at the cost of huge computational complexity, which increases exponentially with the number of users sharing the same channel.

Under multiuser detection perspective the primary concern to increase system capacity consists in achieving high performance with relative low complexity increment. Since the maximum-likelihood (ML) approach is prohibitive for large systems i.e., large number of users sharing the same spectrum in a DS-CDMA systems, the analysis of suboptimal MuD and the respective complexity aspects is of paramount interest. As a result, in the last two decades, a variety of promising multiuser detectors with low complexity and sub-optimum performance were proposed: from the linear detectors, subtractive interference cancelling [1, 2] approaches, to sphere decoding, semidefinite programming (SDP) [3], and more recently heuristic methods. The latter methods have been used for solving different detection models and obtaining near-maximum likelihood (near-ML) performance at cost of polynomial computational complexity. Among heuristic multiuser detection methods are included evolutionary programming (EP) [4], specially the genetic algorithm (GA) [5, 6], particle swarm optimization (PSO) [7, 8, 9], and the deterministic local search (LS) methods [10, 11, 12, 13, 14].

The suboptimal multiuser detection based on semidefinite relaxation (SDR-MuD) [15, 16] and heuristic MuDs [5, 6, 11] approaches were initially proposed to work on low-order modulation, usually BPSK/QPSK signals. For instance, in SDR-MuD, the optimal maximum likelihood (ML) detection problem is carried out by relaxing the associated combinatorial programming problem into an SDP problem with both the objective and the constraint functions being convex functions of continuous variables. Heuristic, deterministic local search methods, and semidefinite relaxation approaches have been shown to yield near-optimal performance under BPSK/QPSK modulation formats [3, 15, 16, 5, 6, 11].

In fourth generation (4G) wideband wireless communication systems, multiple-input-multiple-output (MIMO) schemes and high-order modulation formats, such as $M$-ary phase shift keying ($M$-PSK) or $M$-ary quadrature amplitude modulation ($M$-QAM), are often adopted in order to increase the throughput or the system capacity. Therefore, there has been recently much interest in extending the near-optimum low-complexity detection approaches to detect high-order modulated signals [10, 17, 18, 19, 7, 9, 20, 21, 22, 23]. For instance, SDR-MuD approaches, previously applied to single-input-single-output 64-QAM multicarrier CDMA systems [18], were applied to high-order QAM constellations in MIMO systems in [17, 19, 20, 21], with $M$-PSK constellation in [22], and in coded MIMO systems with pulse-amplitude modulation (PAM) constellations were investigated in [23].

Although the $M$-ary combinatorial problem associated with the optimal $M$-QAM ML detection can be solved by SDP-MuD relaxation methods, simulation results show that, when $M$ is relatively large ($M \geq 32$), the performance...
obtained with SDP relaxation methods becomes far from the ML performance. The reason is that the detection error probabilities of the sequence of binary variables associated with each QAM symbol are unbalanced. Some of the binary variables are more robust to detection errors than others, and recently different strategies has been investigated to exploit this characteristic. In one of them, the strategy consists in detecting those binary variables which are more robust to detection errors first [21], and, with a multistage approach, decisions are made successively, only considering those binary variables which can be detected with higher accuracy in each stage. Based on these decisions, the original problem is reduced to a smaller-sized SDP problem for the undetermined binary variables. This process continues until all binary variables are determined. However, the drawback of this approach is an unavoidable increasing on the multiuser detector complexity associated with a delay increment to detect the symbol of all users.

Finally, heuristic procedures can be adopted in order to achieve good figure of merit in multiuser detection problem, accomplishing excellent performance × complexity trade-offs. Embracing heuristic procedures enable to achieve near-optimum low-complexity multiuser detection under all different scenarios, from low to high-order symbol modulation schemes, combined with fading and multiple input multiple output channels. The local search is an optimization method that consists of searches in a previously established neighborhood [14]. It is a quite simple search method. For the local search algorithm it is important to restrict the neighborhood and to choose a good start point in order to find a valid solution with low complexity. The degree $k$ in the $k$-opt local search ($k$-opt LS) can be chosen taken into account all the $m$ bits mapping a symbol, and evaluating all $k$ Hamming distance candidate-vectors from the current solution that belongs to a constellation of size $M = 2^m$ symbols. It is naturally advantageous to use Gray mapping to enumerate all the $k$—Hamming candidate-vectors. The search complexity increases exponentially with the degree $k$, becoming too computationally expensive to adopt $k \geq 2$ for high-order modulation ($M \geq 8$) with large number of users. In [11], the SISO DS-CDMA MuD problem with BPSK modulation had been solved efficiently using the 1-opt LS. But when the modulation order increases, the $k$-opt LS approach becomes inefficient, suffering of lack of diversification. As a consequence, there are a performance degradation and an increasing in complexity [10].

In the 1-opt LS multiuser detector, all unitary Hamming distance candidate-vectors from the current best solution (obtained from previous iterations) are evaluated. If a better candidate-vector is found, the current best solution is updated, and a new iteration takes place. The search terminates when there is no improvement. Basically, three advantages make the 1-opt LS algorithm a natural choice in order to reach an efficient solution for the MuD problem under BPSK and QPSK modulation formats: a) absence of input parameters; b) simple stop criterion, avoiding prior calculation; c) simple strategy, low complexity with possible additional simplifications [12, 11].

However, there are few works dealing with complex and realistic system configurations. High-order modulation heuristic MuDs in SISO or MIMO systems were previously addressed in [7, 9, 10]. In [10] a heuristic technique was applied to near-optimum asynchronous DS-CDMA multiuser detection problem under 16−QAM modulation and SISO multipath channels. Previous results on literature [11, 12, 8, 4, 13] suggest that evolutionary algorithms and particle swarm optimization have similar performance, and that a simple local search heuristic optimization is enough to solve the MuD problem with low-order modulation (BPSK [11] and QPSK). However, for high-order modulation formats, the LS-MuD does not achieve good performances due to a lack of search diversity, whereas the PSO-MuD has been shown to be more efficient for solving the optimization problem under $M$-QAM modulation [10].

This chapter is organized as follows. The DS-CDMA system model, with description of the optimum detection metric, considering low and high order modulation, single- and multi-path, and single-input-multiple-output channels is revised in Section 2. Additionally, decoupling log-likelihood function (LLF) containing only real values suitable for high-order modulation formats, a revision and contextualization of the swarm intelligence method applied to the multiuser detection, and single/multiple-objective optimization approaches for MuD problem as well, are addressed in this section. Section 3 presents the PSO multiuser detector model. The swarm optimization procedures, including...
an extensive analysis on the PSO input parameters optimization are carried out in Section 4. Exhaustive numerical analysis for several figures of merit under different channel and system scenarios, is carried out in Section 5. In order to evaluate the feasibility of the swarm heuristic technique in solving efficiently the MuD problem, a complexity analysis, in terms of number of operations, is pointed out in Section 6. Finally, the main conclusions of this chapter are highlighted in Section 7.

2 SYSTEM MODEL

In this Section, a single-cell DS-CDMA system model is described for AWGN, flat Rayleigh and multipath Rayleigh channels, considering different modulation schemes, such as binary/quadrature phase shift keying (BPSK/QPSK), and 16-quadrature amplitude modulation (16-QAM), and single or multiple antennas at the base station receiver (uplink).

After describing the conventional detection approach, based on matched filter bank followed by a maximum ratio combining (MRC) rule, subsequent subsection explains how to apply the maximum-likelihood approach through the OMuD in order to achieve the best known uncoded detection performance at cost of a huge complexity increment.

The heuristic multiuser detection model, with single- and multiple objective optimization approaches, is discussed in subsequent subsections as a promising alternative to reach the near-optimum detection performance with much lower complexity than OMuD.

2.1 DS-CDMA

In this section, a single-cell asynchronous multiple access DS-CDMA system model with high order modulation and multiple antennas at the base station receiver (reverse link) is described under selective fading channels as well. Hence, the system model is generic enough to allow describing additive white Gaussian noise (AWGN), flat or synchronous channels, binary modulation formats and single-antenna receiver, as particular cases of the system model discussed in the sequel.

The base-band transmitted signal of the $k$th user is described as [24]

$$s_k(t) = \sqrt{\frac{E_k}{T}} \sum_{i=-\infty}^{\infty} d_k^{(i)} g_k(t - iT),$$

where $E_k$ is the symbol energy, and $T$ is the symbol duration. Each symbol $d_k^{(i)}$, $k = 1, \ldots, K$ is taken independently and with equal probability from a complex alphabet set $A$ of cardinality $M = 2^m$ in a squared constellation points, i.e., $d_k^{(i)} \in A \subset \mathbb{C}$, where $\mathbb{C}$ is the set of complex numbers. Figure 1 shows the modulation formats considered.

The normalized spreading sequence for the $k$-th user is given by

$$g_k(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_k(n) p(t - nT_c), \quad 0 \leq t \leq T,$$

where $a_k(n)$ is a random sequence with $N$ chips assuming the values $\{\pm 1\}$, $p(t)$ is the pulse shaping, assumed rectangular with unitary amplitude and duration $T_c$, with $T_c$ being the chip interval. The processing gain is given by $N = T/T_c$, as illustrated in Figure 2.

![Figure 1: Some modulation formats with Gray mapping.](image-url)
Generally, a slow and frequency selective channel\(^{2}\) is assumed. The expression in (3) is quite general and includes some special and important cases: if \(Q = 1\), a SISO system is obtained; if \(L = 1\), the channel becomes non-selective (flat) Rayleigh; if \(h_{q,k,t}^{(i)} = 1\), it results in the AWGN channel; moreover, if \(\tau_{q,k,t} = 0\), a synchronous DS-CDMA system is characterized.

At the base station, the received signal passes through a matched filter bank (CD), with \(D \leq L\) branches (fingers) per antenna of each user. When \(D \geq 1\), CD is known as Rake receiver. Assuming perfect carrier phase estimation, after despreading the resultant signal is given by

\[
y_{q,k,t} = \frac{1}{T} \int_{t}^{(i+1)T} r(t) g_k(t - \tau_{q,k,t}) dt = A_k h_{q,k,t}^{(i)} \psi_{q,k,d}^{(i)} + S I_{q,k,t}^{(i)} + I_{q,k,t}^{(i)} + \tilde{\eta}_{q,k,t}^{(i)}.
\]

The first term is the signal of interest, the second corresponds to the self-interference (SI), the third to the multiple-access interference (MAI) and the last one corresponds to the filtered AWGN.

To elaborate more about (5), the SI on the \(\ell\)-th correlator output of the \(k\)-th user from \(q\)-th user is given by

\[
S I_{q,k,t}^{(i)} = \frac{A_k}{T} \sum_{d=1}^{D} e^{\varphi_{q,k,d}} \int_{0}^{T} h_{q,k,d}^{(i,\varphi)} g_k(t - \tau_{q,k,t}) g_k(t - \tau_{q,k,d}) dt,
\]

where \(\varphi_{q,k,d} = \varphi_{q,k,t} - \varphi_{q,k,\ell}\) is the relative phase between the carriers of desired and self interfering signals at \(q\)-th receive antenna, and the index \((i,\varphi)\) associated to the symbol of interest \(d_{q,k}^{(i,\varphi)}\) represents the \(i\)-th, or \((i-1)\)-th or even \((i+1)\)-th symbol, depending on the relative delay among the desired (index \(\ell\)) and self interfering (index \(d\)) signals. Besides, when slow (or very slow) fading channels are assumed, the complex channel coefficients \(h_{q,k,d}^{(i,\varphi)}\) hold fixed at least for a couple of symbol period, hence the short time dependence could be eliminated, resulting in \(h_{q,k,d}^{(i,\varphi)} = h_{q,k,d}^{(i)}\).

Similarly to SI, the MAI term on the \(\ell\)-th correlator of the \(k\)-th user, at \(q\)-th receive antenna, is given by

\[
r_{q,k,\ell}^{(i)} = \frac{1}{T} \sum_{u=1, u \neq k}^{K} A_u \sum_{d=1}^{D} e^{\varphi_{q,u,d}} \int_{0}^{T} h_{q,u,d}^{(i,\varphi)} g_k(t - \tau_{q,u,d}) g_k(t - \tau_{q,k,\ell}) dt.
\]

\(^{2}\)Slow channel: channel coefficients were admitted constant along the symbol period \(T\); and frequency selective condition is hold: \(\frac{1}{T} \gg (\Delta B)_{c}\), the coherence bandwidth of the channel.
2 SYSTEM MODEL

with \( \tilde{\varphi}_{q,u,d} = \varphi_{q,u,d} - \varphi_{q,k,l} \). The cross-correlation element between the \( k \)th user, \( \ell \)th path and \( u \)th user, \( d \)th path, at \( q \)th receive antenna, can be identified as

\[
\rho_{k,\ell,u,d}^q = \frac{1}{T} \int_0^T g_k(t - \tau_{q,k,l}) g_u(t - \tau_{q,u,d}) dt. \tag{8}
\]

Considering a maximal ratio combining (MRC) rule\(^3\) with diversity order equal to \( DQ \) for each user, the \( M \)-level complex decision variable is given by

\[
\hat{c}_{k}^{(i)} = \frac{\sum_{q=1}^{Q} \sum_{\ell=1}^{D} \bar{g}_{q,k,\ell} \cdot w_{q,k,\ell}^{(i)}}{K} \quad k = 1, \ldots, K, \tag{9}
\]

where the MRC weights \( w_{q,k,\ell}^{(i)} = \bar{g}_{q,k,\ell} e^{-j\tilde{\varphi}_{q,k,\ell}} \), with \( \bar{g}_{q,k,\ell} \) and \( \tilde{\varphi}_{q,k,\ell} \) being a channel amplitude and phase estimation, respectively. For EGC rule, the channel amplitude weights results all identical\(^4\), but the channel phases need to be estimated yet, \( w_{q,k,\ell}^{(i)} = e^{-j\tilde{\varphi}_{q,k,\ell}} \).

After that, at each symbol interval, decisions are made on the in-phase and quadrature components\(^5\) of \( \hat{c}_{k}^{(i)} \) by scaling it into the constellation limits obtaining \( c_{k}^{(i)} \), and choosing the complex symbol with minimum Euclidean distance regarding the scaled decision variable. Alternatively, this procedure can be replaced by separate \( \sqrt{M} \)-level quantizers \( \text{qtz} \) acting on the in-phase and quadrature terms separately, such that

\[
\hat{s}_{k}^{(i),\text{CD}} = \text{qtz}_{\mathbb{A}_{\text{real}}} \left( \Re \left\{ \hat{c}_{k}^{(i)} \right\} \right) + j \text{qtz}_{\mathbb{A}_{\text{imag}}} \left( \Im \left\{ \hat{c}_{k}^{(i)} \right\} \right), \quad k = 1, \ldots, K, \tag{10}
\]

where \( \mathbb{A}_{\text{real}} \) and \( \mathbb{A}_{\text{imag}} \) is the real and imaginary value sets, respectively, from the complex alphabet set \( \mathbb{A} \), and \( \Re \{ \cdot \} \) and \( \Im \{ \cdot \} \) representing the real and imaginary operators, respectively.

Figure 3 illustrates the general structure of signals for SIMO uplink multi-path \( M \)-QAM DS-CDMA system with MRC Rake receiver.

\(^{3}\)In the absence of interference, MRC is the optimal combining scheme (regardless of fading statistics) but comes at the expense of complexity increasing, since it requires knowledge of all channel fading parameters. Since knowledge of channel fading amplitudes is needed for MRC, this scheme can be used in conjunction with unequal energy signals, such as \( M \)-QAM [25]. Other combining rules include: equal gain combining (EGC), selected combining (SC), switched combining, empirical combining (EC) and hybrid schemes.

\(^{4}\)However, the signal-noise ratio (SNR) relation among EGC and MRC always holds, \( \text{SNR}_{\text{EGC}} < \text{SNR}_{\text{MRC}} \).

\(^{5}\)Note that, for BPSK, only the in-phase term is presented.

Figure 3: Uplink base-band DS-CDMA system model with Conventional receiver. a) \( K \) users transmitters; b) SIMO channel and Conventional receiver with \( Q \) multiple receive antennas.
2.2 Optimum Detection

The optimum maximum likelihood multiuser detector was introduced in 1984 by S. Verdú [2] and is based on log-likelihood function. The OMultiD estimates the symbols for all \( K \) users by choosing the symbol combination associated with the minimal distance metric among all possible symbol combinations in the \( M = 2^m \) constellation points.

In the asynchronous multipath channel scenario, the one-shot asynchronous channel approach is adopted, where a configuration with \( K \) asynchronous users, \( I \) symbols and \( D \) branches is equivalent to a synchronous scenario with \( KID \) virtual users.

Furthermore, in order to avoid handling complex-valued variables in high-order squared modulation formats, henceforward the alphabet set is re-arranged as \( A_{\text{real}} = A_{\text{imag}} = \mathcal{Y} \subset Z \) of cardinality \( \sqrt{M} \), e.g., 16-QAM \((m = 4)\):

\[
d_k^{(i)} \in \mathcal{Y} = \{\pm1, \pm3\}.
\]

The OMultiD is based on the maximum likelihood criterion [2] that chooses the vector of symbols \( \mathbf{d}_p \), formally defined in (16), which maximizes the metric

\[
\mathbf{d}_p^{\text{opt}} = \arg \max_{\mathbf{d}_p \in \mathbb{Y}^{KID}} \{ f(\Omega(\mathbf{d}_p)) \},
\]

where \( f(\Omega(\cdot)) \) is a single or multi-objective function (see Subsection 2.4) that takes into account some combination rule over \( Q \) receive signals, considering the basic log-likelihood function, to be defined formally in (14). In general form, the multi-objective cost function is defined as:

\[
f(\mathbf{d}_p) = [f_1(\mathbf{d}_p), \ldots, f_Q(\mathbf{d}_p)] = [\Omega_1(\mathbf{d}_p), \ldots, \Omega_Q(\mathbf{d}_p)],
\]

where \( Q \) log-likelihood objects are separately applied to the \( p \)th vector-candidate \( \mathbf{d}_p \). On the other hand, in a SIMO channel, the single-objective function generally is written as a combination of the LLFs from all receive antennas, given by

\[
f[\Omega(\mathbf{d}_p)] = \sum_{q=1}^{Q} \Omega_q(\mathbf{d}_p).
\]

Assuming the more general case considered in this chapter, i.e., \( K \) asynchronous users in a SIMO multipath Rayleigh channel with diversity \( D \leq L \), the LLF can be defined as a decoupled optimization problem with only real-valued variables, such that

\[
\Omega_q(\mathbf{d}_p) = 2\mathbf{d}_p^\top \mathbf{W}_q^\top \mathbf{y}_q - \mathbf{d}_p^\top \mathbf{W}_q \mathbf{R} \mathbf{W}_q^\top \mathbf{d}_p,
\]

with definitions

\[
\mathbf{y}_q := \begin{bmatrix} \Re(\mathbf{y}_q) \\ \Im(\mathbf{y}_q) \end{bmatrix}, \quad \mathbf{W}_q := \begin{bmatrix} \Re(\mathbf{A}) - \Im(\mathbf{A}) \\ \Im(\mathbf{A}) \end{bmatrix}, \quad \mathbf{d}_p := \begin{bmatrix} \Re(\mathbf{d}_p) \\ \Im(\mathbf{d}_p) \end{bmatrix}, \quad \mathbf{R} := \begin{bmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix},
\]

where \( \mathbf{y}_q \in \mathbb{C}^{KID \times 1}, \mathbf{W}_q \in \mathbb{R}^{KID \times 2KID}, \mathbf{d}_p \in \mathbb{Y}^{KID \times 1}, \mathbf{R} \in \mathbb{R}^{2KID \times 2KID} \). The vector \( \mathbf{d}_p \in \mathbb{Y}^{KID \times 1} \) in Equation (15) is defined as

\[
\mathbf{d}_p = \begin{bmatrix} (d_1^{(1)} \cdots d_1^{(I)}) \cdots (d_K^{(1)} \cdots d_K^{(I)}) \cdots (d_1^{(I)} \cdots d_1^{(I)}) \cdots (d_K^{(I)} \cdots d_K^{(I)}) \end{bmatrix}^\top.
\]

In addition, the \( \mathbf{y}_q \in \mathbb{C}^{KID \times 1} \) is the despread signal in Equation (5) for a given \( q \), in a vector notation, described as

\[
\mathbf{y}_q = [(y_{q,1,1} \cdots y_{q,1,D}) \cdots (y_{q,K,1} \cdots y_{q,K,D}) \cdots (y_{q,1,1} \cdots y_{q,1,D}) \cdots (y_{q,K,1} \cdots y_{q,K,D})].
\]

Matrices \( \mathbf{H} \) and \( \mathbf{A} \) are the coefficients and amplitudes diagonal matrices, and \( \mathbf{R} \) represents the block-tridiagonal, block-Toeplitz cross-correlation matrix, composed by the sub-matrices \( \mathbf{R}[1] \) and \( \mathbf{R}[0] \), such that [2]

\[
\mathbf{R} = \begin{bmatrix} \mathbf{R}[0] & \mathbf{R}[1]^\top & 0 & \cdots & 0 & 0 \\ \mathbf{R}[1] & \mathbf{R}[0] & \mathbf{R}[1]^\top & 0 & \cdots & 0 \\ 0 & \mathbf{R}[1] & \mathbf{R}[0] & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \mathbf{R}[1] & \mathbf{R}[0] \end{bmatrix},
\]

with \( \mathbf{R}[0] \) and \( \mathbf{R}[1] \) being \( KD \) matrices with elements

\[
\rho_{a,b}[0] = \begin{cases} 1, & \text{if } (k = u) \text{ and } (\ell = l) \\ \rho_{a,l,u,l}^k, & \text{if } (k < u) \text{ or } (k = u \text{ and } \ell < l) \\ \rho_{a,l,u,l}^k, & \text{if } (k > u) \text{ or } (k = u \text{ and } \ell > l), \end{cases}
\]

\[
\rho_{a,b}[1] = \begin{cases} 0, & \text{if } k \geq u \\ \rho_{a,l,k,l}^u, & \text{if } k < u. \end{cases}
\]
where \( a = (k-1)D + \ell \), \( b = (u-1)D + l \) and \( k, u = 1, 2, \ldots, K; \ell, l = 1, 2, \ldots, D; \) the correlation function \( \rho_{k,\ell,u,d}^q \) is calculated through Equation (8).

The evaluation in (11) can either be extended along the whole message, where all symbols of the transmitted vector for all \( K \) users are jointly detected (vector ML approach) or the decisions can be taken considering the optimal single symbol detection of all \( K \) multiuser signals (symbol ML approach). In the synchronous case, the symbol ML approach with \( I = 1 \) is considered, whereas in the asynchronous case the vector ML approach is adopted with \( I = 7 \) (\( I \) must be, at least, equal to three \( (I \geq 3) \)).

The vector \( d_{opt} \) in (15) belongs to a discrete set with size depending on \( M, K, I \) and \( D \). Hence, the optimization problem posed by (11) can be solved directly using a \( m \)-dimensional \( (m = \log_2 M) \) search method. Therefore, the associated combinatorial problem strictly requires an exhaustive search in \( A^{KID} \) possibilities of \( d \), or equivalently an exhaustive search in \( Y^{2KID} \) possibilities of \( d_{opt} \) for the decoupled optimization problem with only real-valued variables. As a result, the maximum likelihood detector has a complexity that increases exponentially with the modulation order, number of users, symbols and branches, becoming prohibitive even for a moderate product values \( mKID \), i.e., even for a BPSK modulation format, medium system loading \( (K/N) \), small number of symbols \( I \) and correlators from \( D \) branches.

### 2.3 Multiuser Detection: A Heuristic Perspective

The maximization of (11) is a combinatorial optimization problem, i.e., the set of possible arguments comprises a finite set. Combinatorial optimization problems can always be solved by exhaustive search, through the cost function computation for every possible argument, following by the selection of the candidate-vector which maximizes the log-likelihood function [2]. OMuD is the exhaustive search realization for the combinatorial multiuser detection problem, which the selection of the optimum vector \( d_{opt} \) can be done in \( O(2^{mKID}) \) operations. The equivalent optimum symbol vector in Equation (11) is estimated in order to maximize the sequence transmission probability given that \( r(t) \) was received, where \( r(t) \) is extended for all message. In [26] it was demonstrated that multiuser detection problem results in a nondeter-

ministic polynomial-time hard (NP-hard) complexity, that is, no algorithm is known for OMuD whose computational complexity results polynomial in terms of the simultaneous number of users \( K \), detected symbols \( I \) and processed branches \( D \), regardless of the cross-correlation matrix, \( R \). Hence, the OMuD is impractical to implement.

Thenceforth, a great variety of suboptimal approaches trying to solve efficiently the MuD problem have been proposed and characterized in the literature: from linear multiuser detectors [2, 27] to heuristic multiuser detectors [28, 5]. Alternatives to OMuD into the class of linear multiuser detectors include the Decorrelator [29], and MMSE [30]. Besides, the classic non-linear multiuser detectors include the interference cancellation (IC) MuD [31] and zero-forcing decision feedback (ZFDF) [32]. The drawback with (non-)linear, ZFDF, and hybrid cancellers sub-optimal MuDs is that they fail in approaching the ML performance under realistic channel and system scenarios while feasibility with relatively low complexity is held.

On the other hand, heuristic near-optimum MuD approaches rise as a promising alternative, saving search-time and complexity. Among them, the genetic algorithm-based multiuser detector (GA-MuD), which was initially proposed in [28] for synchronous AWGN DS-CDMA, the evolutionary programming (EP-MuD) [33], ant colony optimization (ACO-MuD) [34], Tabu search (TS-MuD) [35], local search (LS-MuD) [13], and the particle swarm optimization multiuser detector (PSO-MuD). GA and EP-Muds were extensively compared under different channels scenarios in [6].

For most of the practical cases, MuDs based on heuristic techniques result in almost optimum performance, i.e., very close to the performance reached by the OMuD, with the advantage of smaller computational cost and delay per detected symbol, hence an attractive trade-off between convergence speed, complexity and performance.

The promising characteristics of PSO under multiuser detection problem perspective, such as simplicity regards to GA-MuD approach, while keeping excellent performance×complexity trade-off, have attracted much attention recently on the development of heuristic swarm approach for multiuser detection over a widely practical channel and system scenarios. Thereby, more research
is necessary in order to explore the potentiality of swarm intelligence when applied to solve efficiently the multiuser detection problem.

In the next subsection single-objective optimization and multi-objective optimization approaches, suitable for DS-CDMA systems under SISO and SIMO channels, are described.

## 2.4 Weighting Multi-Objective Optimization

In presence of spatial diversity ($Q$ receive antennas), the multiuser detection problem can be modelled through single or multi-objective optimization (SOO and MOO, respectively) approaches. This subsection brings some arguments in order to adopt SOO or alternatively MOO approach.

Since the maximum-likelihood (or optimum multiuser) detector has a complexity that increases exponentially with the number of users, and therefore it is not practical to be implemented, heuristic approach rise as a possible solution. In the SIMO context, two types of objective function could be considered in order to implement Equation (11). The first one is the linearly combined $Q$-LLFs antenna-diversity-aided strategy (LC $Q$-LLFs), in which each of $K$-bits vector-candidates on the $q$th receive antenna is linearly combined considering all the $Q$ receive antennas. Thus, the fitness value of the $p$th $K$-bits vector-candidate and the associated objective function can be described as [36]

$$f(d_p) = \overline{\Omega}(d_p), \quad \overline{\Omega}(d_p) = \frac{1}{Q} \sum_{q=1}^{Q} \Omega_q(d_p), \quad p = 1, 2, \ldots, P. \quad (20)$$

where $P$ is the population size of PSO strategy. Note that assuming the channel fading associated with different receive antennas been independent, then $\Omega_q(d_p) \neq \Omega_z(d_p)$, for $q \neq z$. As a result, under deep fading condition in some of the $Q$ antennas, the data estimation corresponding to different antennas may result unequal. Note that Equation (13) is a direct realization of the LC $Q$-LLFs strategy.

The second heuristic objective function is a weighting multi-objective (wo) version, suggested in [37], which takes into account independent and combined log-likelihood functions, such that

$$f(d_p) = [\Omega_1(d_p), \ldots, \Omega_Q(d_p), \overline{\Omega}(d_p)] = [f_1(d_p), \ldots, f_Q(d_p), f_{Q+1}(d_p)], \quad p = 1, 2, \ldots, P, \quad (21)$$

where $(Q+1)$ log-likelihood objective functions are separately applied to the $p$th $K$-bits vector-candidate $d_p$ (or equivalently to the $p$th $2K$-bits real-values vector-candidate $d_p$ in Equation (14)). The first $Q$ fitness values are the LLFs of (14), which is related to each of $Q$ receive antennas. The $(Q+1)$th fitness value is the LC $Q$-LLFs given by (20), i.e., $f_{Q+1}(d_p) = \overline{\Omega}(d_p)$. Indeed, due to the independent fading on different receive antennas, in most cases, it is impossible to find a vector-candidate that results the best optimal\(^6\) for $\Omega_q(d_p)$, $\forall q$.

It is clear that the implementation of weighting multi-objective function in (21) implies a significant complexity increment respect to LC $Q$-LLFs objective function, Equation (20). However, numerical results have been indicated that the correspondent performance improvement is only marginal for most of realistic channels and practical system scenarios [37, 38]. Hence, in several situations, the best performance×complexity trade-off lies on a single-objective function described by LC $Q$-LLFs of Equation (20).

In the next section, PSO heuristic approach is explored in order to solve (11) efficiently. Thus, with these $Q+1$ objective functions, the real-values vector-candidates in (14) is explored by the PSO-MuD algorithm, according to the single-objective optimization approach of Equation (20) and, alternatively, can be implemented using the wo $Q$-LLFs optimization approach described by (21).

## 3 PSO MULTIUSER DETECTORS

Particle swarm optimization (PSO) was developed after some researchers have analyzed birds behavior and discern that the advantage obtained through their group life could be explored as a tool for a heuristic search. Considering this

\(^6\)That satisfies simultaneously all LLFs.
new concept of interaction among individuals, in 1995 J. Kennedy and R. Eberhart developed a new heuristic search based on a particle swarm [39].

The PSO principle is the movement of a group of particles, randomly distributed in the search space, each one with its own position and velocity. The position of each particle is modified by the application of velocity in order to reach a better performance [39]. The interaction among particles is inserted in the calculation of particle velocity. In the following subsections, two discrete PSO algorithm versions adapted to the SIMO MuD problem are described in details.

3.1 Discrete Swarm Optimization Algorithm

Discrete or, in several cases, binary PSO [40] is suitable to deal with digital information detection/decoding. Hence, binary PSO is adopted in this chapter.

The PSO’ particle selection for evolving is based on the highest fitness values obtained through (20) or alternatively (21). Decisions under LC Q-LLFs strategy are based on a single entity by combining the information from Q receive antennas. On the other hand, decisions under wQ-LLFs strategy can be established from a single or multiple entities decisions perspective. If the decision rule is based on the highest fitness values from \( Q + 1 \) cost functions, a single-objective weighting optimization approach is obtained. However, considering a subset of \( Q + 1 \) independent solutions to evolve to the next iteration, a multi-objective particle swarm optimization can be applied. In this chapter, only SOO-based decisions procedures were implemented and compared.

So, in order to evaluate (20) or, alternatively (21), over all particle-candidates, it is necessary to calculate the particle velocity and its respective position. For simplicity, in this subsection a SISO channel \((Q = 1 \text{ antenna})\) is assumed. Accordingly, each candidate-vector defined like \( \mathbf{d}_p[t] \) has its binary representation, \( \mathbf{b}_p[t] \), of size \( mKI \), used for the velocity calculation, and the \( p \)th PSO particle position at instant (iteration) \( t \) is represented by the \( mKI \times 1 \) binary vector

\[
\mathbf{b}_p[t] = \left[ b^1_p, b^2_p, \ldots, b^{mKI}_p \right]^\top;
\delta_p = \left[ \delta^1_p, \delta^2_p, \ldots, \delta^mKI_p \right];
\delta^r_{p,v} \in \{0, 1\},
\]

where each binary vector \( b^r_p \) is associated with one \( d_k^{(i)} \) symbol in Equation

\[
(16). \] Each particle has a velocity represented by

\[
v_p[t+1] = \omega \cdot v_p[t] + \phi_1 \cdot U_{p1}[t](b^\text{best}_p[t] - b_p[t]) + \phi_2 \cdot U_{p2}[t](b^\text{best}_g[t] - b_p[t]),
\]

where \( \omega \) is the inertial weight; \( U_{p1}[t] \) and \( U_{p2}[t] \) are diagonal matrices with dimension \( mKI \), whose elements are random variables with uniform distribution \( U \in [0, 1] \); \( b^\text{best}_p[t] \) and \( b^\text{best}_g[t] \) are the best global position and the best local positions found until the \( t \)th iteration, respectively; \( \phi_1 \) and \( \phi_2 \) are weight factors (acceleration coefficients) regarding the best individual and the best global positions influences in the velocity update, respectively.

For MuD optimization with binary representation, each element of \( b_p[t] \) in (23) just assumes “0” or “1” values. Hence, a discrete mode for the position choice is carried out inserting a probabilistic decision step based on threshold, depending on the velocity. Several functions have this characteristic, such as the sigmoid function [40]

\[
S(v^r_{p,v}[t]) = \frac{1}{1 + e^{-v^r_{p,v}[t]}},
\]

where \( v^r_{p,v}[t] \) is the \( r \)th element of the \( p \)th particle velocity vector, \( v^r_p = [v^r_{p,1}, v^r_{p,2}, \ldots, v^r_{p,v,n}] \), and the selection of the future particle position is obtained through the statement

\[
\text{if } u^r_{p,v}[t] < S(v^r_{p,v}[t]), \quad b^r_{p,v}[t + 1] = 1;
\]

\[
\text{otherwise}, \quad b^r_{p,v}[t + 1] = 0,
\]

where \( b^r_{p,v}[t] \) is an element of \( b_p[t] \) (see Equation (22)), and \( u^r_{p,v}[t] \) is a random variable with uniform distribution \( U \in [0, 1] \).

After obtaining a new particle position \( b_p[t + 1] \), it is mapped back into its correspondent symbol vector \( \mathbf{d}_p[t + 1] \), and further in the real form \( \mathbf{d}_p[t + 1] \), for the evaluation of the objective function in (13).

In order to obtain further diversity for the search universe, the \( V_{\max} \) factor is added to the PSO model, Equation (23), which is responsible for limiting the velocity in the range \([\pm V_{\max}]\). The insertion of this factor in the velocity calculation enables the algorithm to escape from possible local optima. The likelihood of a bit change increases as the particle velocity crosses the limits established by \([\pm V_{\max}]\), as shown in Table 1.
Population size $P$ is typically in the range of 10 to 40 [41]. However, considering the multiuser detection problem operating under different optimization scenarios, such as AWGN and multipath channels, high order modulation (M-QAM), and multiple antennas, the size of PSO population must be optimized in order to obtain efficiency under all these different scenarios. Taking into account the background of [8], the PSO’s population size for the MuD problem is set to

$$P = 10 \left[ 0.3454 \left( \sqrt{\pi (mK - 1)} + 2 \right) \right]. \quad (26)$$

Algorithm 1 describes the pseudo-code for the PSO implementation. Next, we discuss the wo Q-LLFs optimization version for the SIMO PSO-MuD.

### 3.2 WO Q-LLF Selection for SIMO PSO-MuD

In the weighting multi-objective PSO (wo-PSO) MuD algorithm, at each iteration the new particle velocity is calculated weighting the contribution of the particle position associated to each receive antenna, based on multi-objective function in (21), resulting

$$v_p[t+1] = \omega \cdot v_p[t] + \sum_{q=1}^{Q+1} \left[ \phi_1 q \cdot U_{p_1}^q [t] \left( b_{p}^{\text{best}, q}[t] - b_p[t] \right) + \phi_2 q \cdot U_{p_2}^q [t] \left( b_{p}^{\text{best}, q}[t] - b_p[t] \right) \right],$$

where now $b_{p}^{\text{best}, q}[t]$ and $b_{p}^{\text{best}, q}[t]$ are the best global and the best local particle positions, respectively, found so far (t-th iteration) in the qth antenna ($q = 1, \ldots, Q$), as well as considering all combined antennas ($q = Q + 1$). Besides, the positive acceleration coefficients satisfy $\sum_{q=1}^{Q+1} \phi_1 q + \phi_2 q = C$, where $C$ is a real constant, generally assumed equal to 4 [42].

In order to obtain fast convergence without losing a certain exploration and exploitation capabilities, $\phi_2$ could be increased, been chosen for the single-carrier BPSK DS-CDMA multiuser detection problem [8] from the range: $\phi_2 \in 1 - S(V_{\text{max}})$.

<table>
<thead>
<tr>
<th>$V_{\text{max}}$</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - S(V_{\text{max}})$</td>
<td>0.2690</td>
<td>0.1192</td>
<td>0.0674</td>
<td>0.0180</td>
<td>0.0067</td>
</tr>
</tbody>
</table>

Table 1: Minimum bit change probability as a function of $V_{\text{max}}$.

### Algorithm 1 PSO Algorithm for the MuD Problem

**Input:** $d^{\text{CD}}, P, G, \omega, \phi_1, \phi_2, V_{\text{max}}$; **Output:** $d$

1. initialize first population: $t = 0$;
   - $B[0] = b^{\text{CD}} \cup \hat{B}$, where $\hat{B}$ contains $(P - 1)$ particles randomly generated;
   - $b_{p}^{\text{best}}[0] = b_p[0]$ and $b_{g}^{\text{best}}[0] = b^{\text{CD}}$;
   - $v_p[0] = 0$: null initial velocity;
2. while $t < G$
   a. calculate $\Omega(d_p[t]), \forall b_p[t] \in B[t]$ using (13);
   b. update velocity $v_p[t]$. $p = 1, \ldots, P$, through (23);
   c. update best positions:
      - for $p = 1, \ldots, P$
         - if $\Omega(d_p[t]) > \Omega(d_{p}^{\text{best}}[t])$, $b_{p}^{\text{best}}[t+1] = b_p[t]$
         - else $b_{p}^{\text{best}}[t+1] = b_{p}^{\text{best}}[t]$
      end
      - if $\exists b_p[t]$ such that $[\Omega(d_p[t]) > \Omega(d_{p}^{\text{best}}[t])] \land 
        [\Omega(d_p[t]) \geq \Omega(d_j[t]), j \neq p],
        b_{g}^{\text{best}}[t+1] = b_p[t]$
      - else $b_{g}^{\text{best}}[t+1] = b_{g}^{\text{best}}[t]$
   d. Evolve to a new swarm population $B[t+1]$, using (25);
   e. set $t = t + 1$.
3. $\hat{b} = b_{g}^{\text{best}}[G]$; $\hat{b} \xrightarrow{\text{map}} \hat{d}$.

$d^{\text{CD}}$: CD output.

$P$: Population size.

$G$: number of swarm iterations.

For each $d_p$, there is a $b_p$ associated.
A comparative study considering combined multi-fitness functions particle swarm optimization (woPSO) and standard PSO S/MIMO multicarrier CDMA MuD, both under single-objective decision approach, was carried out in [37]. In summary, simulation results have been shown the capabilities of both schemes to escape from local solutions, thanks to a balance between exploration and exploitation, resulting similar MuD performance for both approaches.

Note that in (28), the \((Q + 1)\)th cost function and the respective positions \(b^\text{best},Q+1[t]\) and \(b^\text{best},Q+1[t]\) positions allow the exploration capability of the woPSO SIMO, while the others \(Q\) positions, \(b^\text{best},q[t]\) and \(b^\text{best},q[t]\), \(q = 1, \ldots, Q\) bring additional exploitation capability. In order to balance these capabilities, we set \(\phi_1^Q+1 = \phi_2^Q+1 = \frac{1}{2}\) (exploration), and \(\phi_1^Q = \phi_2^Q = \frac{1}{2Q}, 1 \geq q \geq Q\) (exploitation).

Indeed, for each iteration of the woPSO SIMO algorithm, there is a set containing \((Q + 1)\) best particle positions, \(b^\text{best},q[t]\), where the first \(Q\) positions are associated to the \(Q\) end-points of visited Pareto front \(F^*_p\) [43], and the \((Q + 1)\)th one is the position in \(F^*_p\) that maximizes \(\Omega(p)\) in (20). In the same way, there is another set containing \((Q + 1)\) best global positions, \(b^\text{best},q[t]\), where the first \(Q\) positions are \(Q\) end-points of \(F^*_g\), and the \((Q + 1)\)th one is the position in \(F^*_g\) that maximizes \(\Omega(d_p)\).

However, in order to maintain the woPSO optimization process as simple as possible\(^7\), specially in scenarios where the number of receive antennas \((Q)\) increases or, mainly when both system loading \((\frac{K}{N})\) and \(Q\) are large, just the \(Q + 1\) best particles obtained evaluating directly (21) is considered to evolve for the next iteration. After the woPSO iterations terminate, the final estimation vector is determined by the vector \(\hat{b} = b^\text{best},Q+1[G]\), which is associated with the particle position that maximizes \(\Omega(d_p)\). The pseudo-code for the SIMO DS-CDMA strategy is described in Algorithm 2.

---

\(^7\)Note that the algorithm has to be implemented in real time using digital signal processing platforms.

---

**Algorithm 2** woPSO SIMO DS-CDMA

**Input:** \(d^\text{CD}, P, G, \omega, \phi_1, \phi_2, V_{\max};\)  
**Output:** \(\hat{d}\)

1. initialize first population: \(t = 0; v_p[0] = 0;\) null initial velocity;  
   \(B[0] = b^\text{CD} \cup \hat{B},\) with \((P - 1)\) randomly generated;  
   \(b^\text{best},q[0] = b_p[0]\) and \(b^\text{best},q[0] = b^\text{CD}, \forall q.\)

2. while \(t < G\)  
   a. calculate \(\Omega(d_p[t]),\) \(\forall b_p[t] \in B[t],\) Eq. (14), linearly combining the LLF of all antenna according (20);  
   b. update velocity \(v_p[t],\) \(p = 1, \ldots, P,\) Eq. (28);  
   c. update best positions:  
      for \(p = 1, \ldots, P\)  
      for \(q = 1, \ldots, Q\)  
      if \(f_q(d_p[t]) > f_q(d^\text{best},q[t]),\) then \(b^\text{best},q[t + 1] \leftarrow b_p[t]\)  
      else \(b^\text{best},q[t + 1] \leftarrow b^\text{best},q[t].\)  
      end  
      if \(f_{Q+1}(d_p[t]) > f_{Q+1}(d^\text{best},Q+1[t]),\) then \(b^\text{best},Q+1[t + 1] \leftarrow b_p[t];\)  
      else \(b^\text{best},Q+1[t + 1] \leftarrow b^\text{best},Q+1[t].\)  
      end
      end
      end
   d. new swarm population \(B[t + 1],\) Eq. (25);  
   e. set \(t = t + 1.\)

3. \(\hat{b} = b^\text{best},Q+1[G];\) \(\hat{b} \xrightarrow{\text{map}} \hat{d}\).

---

**d**\(^{\text{CD}}\): CD output;  
**P**: swarm population size;  
**G**: number of swarm generations.  
For each \(d_p\), there is a \(b_p\) associated.
4  PSO-MuD PARAMETERS OPTIMIZATION

In this section, the PSO-MuD parameters optimization is carried out in order to improve the PSO algorithm complexity × performance trade-off, allowing fast convergence with relative high quality of the solutions found.

A first analysis of the PSO parameters gives raise to the following behaviors: \( \omega \) is responsible for creating an inertia of the particles, inducing them to keep the movement towards the last directions of their velocities; \( \phi_1 \) aims to guide the particles to each individual best position, inserting diversification in the search; \( \phi_2 \) leads all particles towards the best global position, hence intensifying the search and reducing the convergence time; \( V_{\text{max}} \) inserts perturbation limits in the movement of the particles, allowing more or less diversification in the algorithm.

For the MuD application, the parameters of PSO can be optimized according to the considered system. The capacity of changing the parameters individually is an advantage, since the algorithm can be easily adjusted to suit problems requiring more or less diversification and intensification. Numerical results obtained from Monte-Carlo simulations show that PSO is sensitive to system conditions, and some particular concerns must be taken into account according to the DS-CDMA communication system and channel scenarios.

Recent published works applying PSO to MuD usually assumes conventional values for PSO input parameters, such [44], or optimized values only for specific system and channel scenarios, such [8] for flat Rayleigh channel, [10] for multipath and high-order modulation, and [45] for multicarrier CDMA systems as well. In the following, a survey on the optimization process, displaying the general behavior of PSO and considering various system and channel scenarios is exhibited. Furthermore, the PSO is optimized also for systems with high-order modulation. For all analysis, the initial velocity of all particles is null, i.e., \( v[0] = 0 \).

4.1 \( V_{\text{max}} \) Optimization

**AWGN channels:** Figure 4 indicates that the performance is unchangeable for \( V_{\text{max}} \geq 2 \). However, for an increasing loading, the adoption of \( V_{\text{max}} = 3 \) results in convergence problems as well (nor shown here). Analyzing Table 1, the higher the number of users in the system, the bigger the average number of bit changes, which can result slow convergence. Hence, a good choice is \( V_{\text{max}} = 4 \), corroborating with previous literature results [42].

![Figure 4: \( V_{\text{max}} \) optimization in AWGN channel with BPSK modulation. Average BER × \( V_{\text{max}} \) for \( E_b/N_0 = 7 \) dB, \( K = 15, \phi_1 = 6, \phi_2 = 1, \omega = 1 \), and \( G = 40 \).](image)

**Rayleigh channels.** In [8], the best performance × complexity trade-off for BPSK PSO-MuD algorithm was obtained setting \( V_{\text{max}} = 4 \). Herein, Figure 5 shows that the convergence is already achieved for \( V_{\text{max}} > 2 \) in a medium loading condition. However, to avoid lack of intensification for full loading, \( V_{\text{max}} = 4 \) is adopted as a good alternative.

4.2 \( \phi_1 \) and \( \phi_2 \) Optimization

**AWGN channels:** Under AWGN channels, two situations are verified regarding \( \phi_1 \): for high values of \( \phi_1 \), the algorithm tends to converge slowly, but achieving better performance. Values lower than 3 results in a better convergence speed, but with a poor performance, as can be seen in Figure
6. Moreover, the performance gap becomes evident with an increase in the number of users.

Considering $\phi_2$, Figures 6 and 7 show different convergence and performance behavior when $\phi_2$ increases over the interval $[1; 10]$. Figure 7.(a) shows that the performance of PSO-MuD degrades with the increasing number of users, being more evident for high values of $\phi_2$. Instead of the large number of required iterations for convergence with $\phi_2 = 1$, Figure 7.(b), the performance improvement justifies the adoption of such value. Therefore, for the AWGN channels, a good choice for the acceleration coefficients seems to be $\phi_1 = 6$ and $\phi_2 \in [1; 2]$.

Rayleigh channels: Distinctly from AWGN case, for Rayleigh channels the performance improvement caused by $\phi_1$ increment is no longer evident, and its value can be reduced without performance losses, as can be seen in Figure 8. Therefore, a good choice seems to be $\phi_1 = 2$, achieving a reasonable convergence rate.

Figure 9.(a) illustrates different convergence performances achieved with
4 PSO-MUD PARAMETERS OPTIMIZATION

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Figure 8: $\phi_1$ optimization in flat Rayleigh channels with BPSK modulation, $E_b/N_0 = 22$ dB, $K = 15$, $\phi_2 = 10$, $V_{\text{max}} = 4$, and $\omega = 1$.

$\phi_1 = 2$ and $\phi_2 \in [1; 15]$ for medium system loading and medium-high $E_b/N_0$. Even for high system loading, the PSO performance is quite similar for different values of $\phi_2$, as observed in Figure 9 (b). Hence, considering the performance x complexity trade-off, a reasonable choice for $\phi_2$ under flat Rayleigh channels is $\phi_2 = 10$.

4.3 $\omega$ Optimization

It is worth noting that relatively larger value for the inertial weight PSO input parameter, $\omega$, is helpful for global optimum, and lesser influenced by the best global and local positions, while a relatively smaller value for $\omega$ is helpful for course convergence, i.e., smaller inertial weight encourages the local exploration [41, 46] as the particles are more attracted towards $\mathbf{b}_{p}^{\text{best}}[t]$ and $\mathbf{b}_{g}^{\text{best}}[t]$.

**AWGN channels**: Figure 10 offers some insight on parameter $\omega$ optimization process under AWGN channels. It is observed that the adoption of high values of $\omega$ implies in fast convergence, but this means a lack of search diversity, and the algorithm can easily be trapped in some local optimum, whereas small values for $\omega$ result in slow convergence due to excessive bit changes. The optimized value is $\omega = 1$, which achieves a better performance x complexity, as shows Figure 10.

**Flat channels**: Similarly to AWGN case, Figure 11 shows the convergence of the PSO scheme for different values of $\omega$. It is evident that the best performance x complexity trade-off is accomplished with $\omega = 1$.

Many research papers have been proposed new strategies for PSO principle in order to improve its performance and reduce its complexity, for instance, in [47] the authors have been discussed adaptive nonlinear inertia weight in order to improve PSO convergence. However, the results obtained by PSO for both channels show that no further specialized strategy is necessary, since the conventional PSO works well to solve the MUD DS-CDMA problem in several practical scenarios.
4 PSO-MUD PARAMETERS OPTIMIZATION

4.4 $\phi_1$ and $\phi_2$ Optimization under High-order Modulation

The optimization process for systems with high-order modulation is quite similar compared to BPSK process. Only the case with Rayleigh channel is evaluated, since it represents a more realistic condition. Special attention is given for $\phi_1$ and $\phi_2$ optimization, which are different from values obtained with BPSK modulation.

The optimization of the inertial weight and maximum velocity achieve analogous results, and here their optimized values, obtained by simulation, are $\omega = 1$ and $V_{\text{max}} = 4$ for both QPSK and 16-QAM modulations. Under flat Rayleigh channels, and with these $\omega$ and $V_{\text{max}}$ optimized values, the acceleration factors could be optimized for QPSK modulation (Figure 12) and 16-QAM (Figure 13). Again, under 16-QAM modulation, the PSO-MuD requires more intensification, once the search becomes more complex due to each symbol to contain 4 bits. Figure 13 shows the convergence curves for different values of $\phi_1$ and $\phi_2$. The performance gap is more evident with an increasing number of users and $E_b/N_0$. Analyzing this result, the chosen values are $\phi_1 = 6$ and $\phi_2 = 1$.

4.5 Optimization for Systems with Diversity Exploration

The best range for the acceleration coefficients under resolvable multipath channels ($L \geq 2$) for MuD SISO DS-CDMA problem seems $\phi_1 = 2$ and $\phi_2 \in [12; 15]$, as indicated by the simulation results shown on Figure 14. For medium system loading and signal-noise ratio (SNR) as well, Figure 14 indicates that the best values for acceleration coefficients are $\phi_1 = 2$ and $\phi_2 = 15$, allowing the combination of fast convergence and near-optimum performance achievement.

4.6 Optimized parameters for PSO-MuD

As mentioned previously, the optimized input parameters for PSO-MuD vary regarding the system and channel scenario conditions. Monte-Carlo simula-

Figure 10: $\omega$ optimization under AWGN channel with BPSK modulation, considering $E_b/N_0 = 7$ dB, $K = 15$, $\phi_1 = 6$, $\phi_2 = 2$, and $V_{\text{max}} = 4$.

Figure 11: $\omega$ optimization under flat Rayleigh channels with BPSK modulation, $E_b/N_0 = 22$ dB, $K = 15$, $\phi_1 = 2$, $\phi_2 = 10$ and $V_{\text{max}} = 4$. 
Figure 12: $\phi_1$ and $\phi_2$ optimization under flat Rayleigh channels for QPSK modulation, $E_b/N_0 = 22$ dB, $K = 15$, $\omega = 1$ and $V_{\text{max}} = 4$.

Figure 13: $\phi_1$ and $\phi_2$ optimization under flat Rayleigh channels for 16-QAM modulation, $E_b/N_0 = 30$ dB, $K = 15$, $\omega = 1$ and $V_{\text{max}} = 4$.

Figure 14: $\phi_1$ and $\phi_2$ optimization under Rayleigh channels with path diversity ($L = D = 2$) for BPSK modulation, $E_b/N_0 = 22$ dB, $K = 15$, $\omega = 1$, $V_{\text{max}} = 4$.

Table 2: Optimized parameters for asynchronous PSO-MuD.

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<th>Channel</th>
<th>Modulation</th>
<th>$\mathcal{L}$ range</th>
<th>$\omega$</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$V_{\text{max}}$</th>
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<td>Flat Rayleigh</td>
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<td>2</td>
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</tr>
<tr>
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</table>
5 NUMERICAL RESULTS

In this section, the PSO multiuser detector performance is analyzed in terms of average bit- or symbol-error-rate (BER\textsubscript{Avg} or SER\textsubscript{Avg}), via Monte-Carlo simulation (MCS). Several aspects are considered, such as high order modulation, perfect and imperfect channel estimation at the receiver side, NFR effect, and the impact of spatial and multipath diversity order. The results obtained are compared with theoretical single-user bound (SuB), according to Appendix A, since the OMuD computational complexity results prohibitive.

Monte-Carlo Simulation setup is summarized in the Appendix B. The general system and channel parameters adopted in this chapter are described in the sequel.

The analysis presented in the sequel is divided according to system and channel conditions as follows:

- AWGN channel with BPSK modulation;
- flat Rayleigh and multipath channels with BPSK modulation, including space or time diversity;
- flat Rayleigh channel with QPSK/16-QAM modulation and space diversity;

In order to evaluate the impact of realistic limitations over a DS-CDMA system, four variety of simulation results are discussed in the subsequent analysis of the PSO-MuD:

- Convergence curve as a function of number of iterations;
- BER/SER performance as a function of $E_b/N_0$;
- BER/SER performance as a function of number of users $K$, i.e., robustness against system loading increasing ($\mathcal{L} = K/N$), and;
- BER/SER performance as a function of the increasing in the amount of power interference, i.e., robustness against NFR.

### 5.1 AWGN Channels

System conditions and PSO parameters for AWGN channel simulations are summarized in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Adopted Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS-CDMA System</td>
<td></td>
</tr>
<tr>
<td># Rx antennas</td>
<td>$Q = 1$</td>
</tr>
<tr>
<td>Spreading Sequences</td>
<td>Random, $N = 31$</td>
</tr>
<tr>
<td>modulation</td>
<td>BPSK</td>
</tr>
<tr>
<td># mobile users</td>
<td>$K \in [5, 30]$</td>
</tr>
<tr>
<td>Received SNR</td>
<td>$E_b/N_0 \in [2; 8]$ dB</td>
</tr>
<tr>
<td>PSO-MuD Parameters</td>
<td></td>
</tr>
<tr>
<td>Population size, $\mathcal{P}$</td>
<td>Eq. (26)</td>
</tr>
<tr>
<td>acceleration coefficients</td>
<td>$\phi_1 = 6; \phi_2 = 1$</td>
</tr>
<tr>
<td>inertia weight</td>
<td>$\omega = 1$</td>
</tr>
<tr>
<td>Maximal velocity</td>
<td>$V_{\text{max}} = 4$</td>
</tr>
</tbody>
</table>

Although DS-CDMA systems have been equipped with a power control scheme, the fast variations intrinsic to the wireless channel may result in different received power among the users. In such case, the detection of the weak users is affected and therefore their performance is degraded. Figure 15 shows that the PSO-MuD is robust to the NFR, keeping its performance and convergence rate almost constant, while the CD is strongly degraded under $NFR > 0$.

Besides, once the DS-CDMA system is limited by multiple access interference and the CD does not eliminate this MAI, its performance does not reach the SuB of the channel and it is expected that the correspondent performance gap regards to SuB becomes bigger with the $E_b/N_0$ increasing. Figure 16 shows this behavior for the CD, while the PSO-MuD performance remains close to the SuB performance, independently of the received $E_b/N_0$.

When the number of users sharing the system increases, the multiple access interference becomes larger and the CD performance is strongly degraded, since it is not able to eliminate this MAI. Figure 17 shows the increasing MAI...
5 NUMERICAL RESULTS

Figure 15: Convergence performance of PSO-MuD in AWGN channel and BPSK modulation, for $K = 15$ and $E_b/N_0 = 7$ dB, with (a) perfect power control; (b) interferes with $NFR = 8$ dB. The performance exhibited in (b) considers only the weaker users.

Figure 16: Performance of PSO-MuD under AWGN channels for medium system loading ($K = 15$, $N = 31$) and BPSK modulation.

5.2 Rayleigh Channels

The adopted PSO-MuD parameters, as well as system and channel conditions employed in Monte Carlo simulations are summarized in Table 4. Non-line-of-sight propagation (Rayleigh) channels are adopted, including flat ($L = 1$) and
selective channels with $L = 2$ or $L = 3$ independent paths.

Table 4: System, channel and PSO-MuD parameters for fading channels performance analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Adopted Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DS-CDMA System</strong></td>
<td></td>
</tr>
<tr>
<td># Rx antennas</td>
<td>$Q = 1, 2, 3$</td>
</tr>
<tr>
<td>Spreading Sequences</td>
<td>Random, $N = 31$</td>
</tr>
<tr>
<td>modulation</td>
<td>BPSK, QPSK and 16-QAM</td>
</tr>
<tr>
<td># mobile users</td>
<td>$K \in [5; 31]$</td>
</tr>
<tr>
<td>Received SNR</td>
<td>$E_b/N_0 \in [0; 30]$ dB</td>
</tr>
<tr>
<td><strong>PSO-MuD Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Population size, $P$</td>
<td>Eq. (26)</td>
</tr>
<tr>
<td>acceleration coefficients</td>
<td>$\phi_1 = 2, 6; \phi_2 = 1, 10$</td>
</tr>
<tr>
<td>inertia weight</td>
<td>$\omega = 1$</td>
</tr>
<tr>
<td>Maximal velocity</td>
<td>$V_{\text{max}} = 4$</td>
</tr>
<tr>
<td><strong>Rayleigh Channel</strong></td>
<td></td>
</tr>
<tr>
<td>Channel state info. (CSI)</td>
<td>perfectly known at Rx</td>
</tr>
<tr>
<td>Number of paths</td>
<td>$L = 1, 2, 3$</td>
</tr>
</tbody>
</table>

The results presented for AWGN show that the PSO-MuD performance is almost constant while varying the number of users, but it is expected that its speed of convergence become as slower as higher the number of users. Considering the PSO-MuD in a flat Rayleigh channel, Figure 18 exhibits this effect, showing first the convergence for a system with $K = 15$ users (half system loading), 18.(a), and second for system with $K = 31$ users (full system loading), 18.(b). Note that there is a little loss in performance after convergence for $L = 1$, due to the PSO-MuD does not totally eliminate the MAI, and its convergence speed is slower, since the search becomes more intricate. However, the PSO-MuD behavior is homogenous, consistent and effective under increasing system loading scenarios.

Likewise to the AWGN case, Figure 19 corroborates the effectiveness of the PSO-MuD in flat Rayleigh channels against the increasing multiple access interference. As can be seen, independently of the loading, PSO-MuD always approaches the SuB performance. It is evident that the conventional detector, based on the matched filter, is very sensitive to the number of users, caused by the direct relation of increasing number of users and MAI.

5.2.1 Path Diversity

DS-CDMA systems are able to explore the path diversity, since the spread of the signal may create distinguishable copies of transmitted signal at the receiver, with distinct delays and channel coefficients [24]. This can be explored, representing a diversity gain and improving the system capacity. Figure 21
Figure 19: Performance of PSO-MuD under flat Rayleigh channels for different system loading, with $E_b/N_0 = 25$ dB and BPSK modulation.

Figure 20: Average BER Avg $\times E_b/N_0$ for flat Rayleigh channel with $K = 15$: (a) perfect power control; (b) $NFR = +6$ dB for 7 users. In scenario (b), the performance is calculated only for the weaker users.

Tables 5:

<table>
<thead>
<tr>
<th>Param.</th>
<th>PD-1</th>
<th>PD-2</th>
<th>PD-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path, $\ell$</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_\ell$</td>
<td>0</td>
<td>0</td>
<td>$T_c$</td>
</tr>
<tr>
<td>$E[\gamma^2]$</td>
<td>1.0000</td>
<td>0.8320</td>
<td>0.1680</td>
</tr>
</tbody>
</table>

Note there is a performance gain with the exploration of such diversity, verified in both the Rake receiver and PSO-MuD. Nevertheless, the Rake is still affected by MAI, Equation (7), and now by the self interference, Equation (6), since other paths interfere in the detection of signals. The PSO-MuD performance is close to SuB in all cases, exhibiting its capability of exploring path diversity and dealing with SI as well. In addition, the convergence aspects are kept for all conditions.

The PSO-MuD single-objective function, Equation (13), takes into account the channel coefficient estimation of each user, and imperfect channel estimation degrades its performance. In order to raise a quantitative characterization of such aspect, the PSO-MuD is evaluate under channel error estimation, which are modelled through the continuous uniform distributions $\mathcal{U}[1 \pm \epsilon]$, centralized on the true values of the coefficients, resulting

$$\gamma^{(i)}_{k,\ell} = \mathcal{U}[1 \pm \epsilon_\gamma] \times \gamma^{(i)}_{k,\ell}, \quad \theta^{(i)}_{k,\ell} = \mathcal{U}[1 \pm \epsilon_\theta] \times \theta^{(i)}_{k,\ell},$$

where $\epsilon_\gamma$ and $\epsilon_\theta$ are the maximum module and phase normalized errors for the channel coefficients, respectively.

For a low-moderate SNR and medium system loading ($L = 15/31$), Figure 22 shows the performance degradation of the PSO-MuD considering BPSK
modulation, flat and selective ($L = 2$ paths) Rayleigh channels with estimation errors of order of 10% or 25%, i.e., $\epsilon_\gamma = \epsilon_\theta = 0.10$ or $\epsilon_\gamma = \epsilon_\theta = 0.25$, respectively. It is observed that when channel estimation presents errors, the performance is degraded, and the higher the SNR, more evident is the gap between PSO-MuD performance and SuB. However the PSO-MuD is still much better than CD in any situation, and one can see that if the estimation error is constrained by $\epsilon_\gamma, \epsilon_\theta < 0.1$, a reasonable performance can be achieved.

As expected from previous results for flat Rayleigh channel, Figure 23 shows the performance degradation CD for $L = 1$ and $L = 2$ as function of number of users $K$, and the robustness of PSO-MuD against the increasing loading. It is evident that the PSO-MuD has a superior performance than CD, for all evaluated loading, being more considerable for full loading and $L = 2$ paths. As can be seen, independently of the loading and number of paths, PSO-MuD always accomplishes a much better performance.
5.2.2 Spatial Diversity

Spatial diversity is an approach which aims to exploit the different correlation between received signals at different locations, so as to avoid deep fading in the received signal. In the results presented here, two assumptions are considered when there are more than one antenna at receiver: first, the average received power is equal for all antennas; and second, the SNR at the receiver input is defined as the received SNR per antenna. Therefore, there is a power gain of 3 dB when adopted $Q = 2$, 6 dB with $Q = 3$, and so on.

Although the objective function calculation becomes more expensive for a larger number of antennas, one can expect that the convergence of PSO-MuD be similar, once the number of detected symbols are equal. The effect of increasing the number of receive antennas in the convergence curves is shown in Figure 24, where PSO-MuD works on systems with $Q = 1$, 2 and 3 antennas. Note that there is an improvement in the performance, given by the diversity and SNR gains, for both CD and PSO-MuD. A delay in the PSO-MuD convergence is observed when more antennas are added to the receiver, caused by the larger gap that it has to surpass. Furthermore, PSO-MuD achieves the SuB performance for all the three cases.

Regarding the effect of channel estimation errors in the performance of the PSO-MuD, Figure 25 shows this effect under similar conditions adopted in Figure 22, but now with spatial diversity, i.e., $Q = 1$, 2 antennas. Note that PSO-MuD reaches the SuB in both conditions with perfect channel estimation, and the improvement is more evident when the diversity gain increases. Note that, in this case, the diversity gain is higher when compared to the multipath case in Figure 22, since the average energy is equally distributed among antennas, while for path diversity is considered a realistic exponential power-delay profile. Moreover, there is a SNR gain of +3 dB in the $E_b/N_0$ for each additional receive antenna. Although there is a general performance degradation when the error in channel coefficient estimation increases, PSO-MuD still achieves much better performance than the CD under any error estimation condition, being more evident for larger number of antennas.

If the number of users sharing the system increases, the MAI substantially increases too, and the performance of single-user detectors, such as conven-
tional detector (CD), is degraded accordingly. Nevertheless, Figure 26 shows that PSO-MuD performance is robust to the system loading, and increases accordingly with the number of receive antennas, while the CD is very sensitive, being the performance gap between them more evident for $Q = 2$ and high loading. Again, the performance difference to Figure 23 is explained by the higher diversity gain (average energy evenly distributed among receive antennas) and the SNR gain, once each antenna has a unitary average received energy.

5 NUMERICAL RESULTS

5.2.3 High-order Modulation

Regarding the new generations of wireless systems, beyond spatial and path diversity, the wireless systems explore high-order modulation formats in order to increase spectral efficiency. Indeed, Figure 27 indicates that PSO-MuD could achieve suitable convergence under several modulation formats and antenna diversity. This Figure exhibits convergence for three different modulations: (a) BPSK, (b) QPSK, and (c) 16-QAM. Note that, the PSO-MuD convergence behavior is quite similar, reaching the SuB in a number of iterations proportional to the order modulation i.e., proportional to the size of search universe. It is worth mentioning, as presented in Table 2, that the PSO-MuD optimized parameters is specific for each order modulation.

Figure 26: PSO-MuD performance × number of users $K$, in flat Rayleigh channel, BPSK modulation, $E_b/N_0 = 15$dB, and spatial diversity.

Figure 27: Convergence of PSO-MuD under flat Rayleigh channel, $E_b/N_0 = 20$ dB, and (a) $K = 24$ users with BPSK modulation, (b) $K = 12$ users with QPSK modulation and (b) $K = 6$ users with 16-QAM modulation.

When a QPSK or higher-order modulation formats are considered, such as $M$-QAM or $M$-PSK with $M \geq 8$, the multiuser detection becomes more complex, since each symbol carries $m = \log_2 M$ bits. This implies that, if there are $K$ users to be jointly detected, PSO-MuD must deal with $m2K$ bits if a decoupled representation and optimization procedure with only real
value variables is adopted, Equation (14), since the PSO algorithm employed here has a binary representation. Hence, this may affect the performance of PSO-MuD. Nevertheless, varying $E_b/N_0$, no performance loss was observed, as shown in Figure 28, for $Q = 1$ and 2 receive antennas and medium system loading. In both receive antenna cases, the achieved performance of PSO-MuD is close to the QPSK SuB. On the other hand, the CD performance is severely affected with order modulation increasing, resulting in poorer performance regarding the BPSK modulation case of Figure 25.

Systems with higher-order modulation tends to be more and more prejudiced by multiple access interference, since the amplitude carries more information. As expected, for 16-QAM modulation, Figure 29 shows the poorer performance of CD regarding the QPSK and BPSK modulation cases. This increasing degradation is mainly caused by distance reduction of the symbols in the constellation when order modulation increases, and by the MAI. However, even so, the performance of the PSO-MuD under medium system loading ($\mathcal{L} = \frac{15}{31}$) reaches the SuB for 16-QAM, in both conditions, with $Q = 1$ and 2, for all evaluated SNR range.

Finally, when the number of users increases, Figure 30 shows that for 16-QAM modulation and flat Rayleigh channels, the PSO-MuD performance degradation with $\phi_1 = 6$, $\phi_2 = 1$ is quite slight in the range ($0 < \mathcal{L} \leq 0.5$), and always much better than CD for all system loading values. The increment in performance degradation when $\mathcal{L} > 0.5$ could be explained by the fact that further input parameter optimization (mainly over $\phi_1$ and $\phi_2$) must be accomplished in order to achieve complete convergence under $P$ population size and $G$ swarm iterations.

**6 COMPLEXITY ANALYSIS**

The analysis of the computational complexity is essential, since the MuD application requires real-time processing and the time requirement is hard. The OMuD complexity, as previously mentioned, grows exponentially with the number of users. Heuristic detectors, such PSO-MuD related here, work in the sense of diminishing this complexity by leading the search to suitable regions of the search space avoiding useless evaluations, achieving polynomial
complexity. In the sequel, an analytical and a numerical analysis is carried out, considering multiplications, additions and random number generations.

6.1 Analytical Complexity

In the sense of fully explore the complexity of MuD detectors, an analysis of the log-likelihood function is presented, followed by its implication on OMuD and PSO-MuD. The LLF considered here needs the correlation matrix, which is calculated in Equation (18). Since this calculation is necessary for both MuDs, it is omitted here. The remain operations are detailed in the following.

6.1.1 OMuD Complexity

The number of operations depends on the number of users $K$, number of symbols $I$, modulation order $m$, number of receive antennas $Q$ and number of paths; and the PSO-MuD performance also depends on the population size $P$ and a priori iterations $G$ for convergence. As mentioned previously, for the OMuD, the number of operations increases exponentially with the number of users. For a generic modulation order $m$, the entire search space is $2^{mKI}$. It is worth mentioning that in the mathematical formulation of LLF, there are null products that can be avoided when considering a DSP implementation, simplifying the evaluation of each candidate-vector. Hence, once one cost function calculation demands $[4 \cdot (2KI)^2 + 5 \cdot (2KI)] \cdot Q$ products, and $[(2KI - 1) \cdot (4KI + 1) + 1] \cdot Q$ additions, and the total complexity of OMuD can be acquired multiplying these values by the size of the search universe of candidate-vectors, which results in

$$
\text{Mult.} : (2^{mKI}) \cdot [4 \cdot (2KI)^2 + 5 \cdot (2KI)] \cdot Q, \\
\text{Add.} : (2^{mKI}) \cdot [(2KI - 1) \cdot (4KI + 1) + 1] \cdot Q.
$$

Nevertheless, according to [11], the cost function calculation can be simplified through a recursive approach when exists a deterministic behavior in the search. Therefore, after the first cost function calculation, a look-up table is used, and the computational complexity of OMuD becomes

$$
\text{Mult.} : (2^{mKI} - 1) \cdot (3 + 4KI) \cdot Q + [4 \cdot (2KI)^2 + 5 \cdot (2KI)] \cdot Q, \\
\text{Add.} : (2^{mKI} - 1) \cdot [(2KI + 4) \cdot Q - 1] + [(2KI - 1) \cdot (4KI + 1) + 1] \cdot Q,
$$

which is much smaller than the previous one and accomplishes identical performance.

6.1.2 PSO-MuD Complexity

In the PSO case, the cost function calculation requires the same number of operation. In addition, PSO evolving is random and simplifications from deterministic aspects presented in OMuD are not applicable here. Further operations are required for PSO principle, in order to population evolving, essentially in the velocity calculation. Considering an a priori number of generations $G$ and a population size $P$, the total number of operations required by the PSO
6 COMPLEXITY ANALYSIS

6.2 Numerical Complexity

From the number of operations presented in the previous subsection, one can conclude that the analytical complexity of OMuD and PSO-MuD differs mainly by the simplifications of the deterministic search and by the number of evaluated candidate-vectors.

The optimum detector evaluates all possible combinations of symbols for all users, implying in the exponential term $2^{mKI}$; however, for each candidate-vector evaluation, simplifications due to its deterministic search are applied.

The PSO-MuD has a polynomial number of cost function calculation, which depends directly on the population size and the number of a priori iterations, but its cost function calculation can not be simplified.

It is expected that the number of operations to be smaller for OMuD for simple and sometimes non-practical detection cases, i.e., with synchronous transmissions, few users and low modulation order, and in absence of diversity exploitation. For more elaborated and realistic DS-CDMA systems, the PSO becomes much simpler regarding the OMuD complexity. In the sequel, a numerical analysis presents the number of operations for different system conditions.

### 6.2.1 AWGN Synchronous Channel

This analysis consists of the simplest case studied in this chapter. The communication channel is only AWGN, and it has neither fading, nor spatial nor path diversities. Moreover, the uplink communication is synchronous, and a considerable simplification is achieved when the LLF presents $I = 1$.

Table 6 shows the number of multiplication, additions and random number generation as a function of $E_b/N_0$. It is possible to see the increment in the number of operations when the $E_b/N_0$ increases, because the number of iterations for convergence increases when the $E_b/N_0$ increases.

A comparison between MuDs shows that, even in a simple channel and system scenarios, the PSO-MuD has a comparable complexity with OMuD, since the search universe of optimization is small in these scenarios. Moreover, it is worth mentioning that PSO-MuD has approximately the same number of multiplications and additions, while for OMuD the number of multiplications is twice the number of additions.

Table 6: Number of operations for system in AWGN synchronous channel for a varying $E_b/N_0$, with $K = 15$ users and BPSK modulation.

<table>
<thead>
<tr>
<th>$E_b/N_0$ [dB]</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mult. [$\times 10^5$]</td>
<td>9.07</td>
<td>9.64</td>
<td>10.77</td>
<td>11.34</td>
<td>12.48</td>
<td>14.74</td>
<td>18.71</td>
<td>20.68</td>
</tr>
<tr>
<td>RNG [$\times 10^4$]</td>
<td>2.11</td>
<td>2.25</td>
<td>2.52</td>
<td>2.65</td>
<td>2.92</td>
<td>3.46</td>
<td>4.41</td>
<td>0</td>
</tr>
</tbody>
</table>

### 6.2.2 Flat Rayleigh Channel

When a more realistic flat Rayleigh channel is considered, the convergence becomes faster than in AWGN channel, and the complexity reduction over OMuD is more evident when the number of users increases. With $K = 15$ users, Table 7 shows the number of operations when the $E_b/N_0$ is varying, similarly to AWGN case, with BPSK modulation, $Q = 1$ antenna and $D = 1$ path.
Table 7: Number of operations for system under flat Rayleigh channel, varying $E_b/N_0$, $K = 15$ users, and BPSK modulation.

<table>
<thead>
<tr>
<th>$E_b/N_0$ [dB]</th>
<th>PSO-MuD</th>
<th>OMuD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.54</td>
<td>4.41</td>
</tr>
<tr>
<td>5</td>
<td>6.81</td>
<td>7.73</td>
</tr>
<tr>
<td>10</td>
<td>7.94</td>
<td>9.39</td>
</tr>
<tr>
<td>15</td>
<td>9.64</td>
<td>8.83</td>
</tr>
<tr>
<td>20</td>
<td>9.07</td>
<td>9.94</td>
</tr>
<tr>
<td>25</td>
<td>10.21</td>
<td>10.49</td>
</tr>
<tr>
<td>30</td>
<td>10.77</td>
<td>10.83</td>
</tr>
</tbody>
</table>

As expected, from simulations results, the total number of operations is hardly affected by the number of users, even for a simple system scenario with no spatial and no path diversities. It is important highlight that for $K > 15$ the computational complexity of OMuD is much higher than PSO-MuD, being prohibitive for real-time implementation. Table 8 shows the number of operations when the number of users is varying, with $E_b/N_0 = 20$ dB, BPSK modulation, $Q = 1$ antenna and $D = 1$ path.

Table 8: Number of operations for system under flat Rayleigh channel for $K$ varying, with $E_b/N_0 = 20$ dB and BPSK modulation.

<table>
<thead>
<tr>
<th>$K$ [users]</th>
<th>PSO-MuD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.46</td>
</tr>
<tr>
<td>10</td>
<td>2.58</td>
</tr>
<tr>
<td>15</td>
<td>13.04</td>
</tr>
<tr>
<td>20</td>
<td>25.91</td>
</tr>
<tr>
<td>25</td>
<td>25.91</td>
</tr>
</tbody>
</table>

6.2.3 Path Diversity

The exploration of path diversity is used in asynchronous systems under selective fading channels, which implies in increasing the receiver complexity, since replicas of received signal and symbols are processed and evaluated in parallel. Table 9 shows the number of operations when $K$ is varying, with $E_b/N_0 = 20$ dB, BPSK modulation, $Q = 1$ antenna and $D = 2$ paths. It is observed when comparing Tables 8 and 9 that the number of operations increases drastically due the diversity exploitation and asynchronous system. Although the number of symbols is three times for the asynchronous case ($I = 3$), the complexity increment is much higher. In the OMuD case, the number of operations is also much larger, specially for a high number of users.

Table 9: Number of operations for system in multipath Rayleigh channel for $K$ varying, with $E_b/N_0 = 20$ dB and BPSK modulation.

<table>
<thead>
<tr>
<th>$K$ [users]</th>
<th>PSO-MuD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mult. [$\times 10^8$]</td>
</tr>
<tr>
<td>5</td>
<td>0.03</td>
</tr>
<tr>
<td>10</td>
<td>0.19</td>
</tr>
<tr>
<td>15</td>
<td>0.57</td>
</tr>
<tr>
<td>20</td>
<td>1.91</td>
</tr>
<tr>
<td>25</td>
<td>3.97</td>
</tr>
<tr>
<td>30</td>
<td>8.27</td>
</tr>
</tbody>
</table>

Table 9: Number of operations for system in multipath Rayleigh channel for $K$ varying, with $E_b/N_0 = 20$ dB and BPSK modulation.

<table>
<thead>
<tr>
<th>$K$ [users]</th>
<th>OMuD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mult. [$\times 10^8$]</td>
</tr>
<tr>
<td>5</td>
<td>0.02</td>
</tr>
<tr>
<td>10</td>
<td>1.32·10^3</td>
</tr>
<tr>
<td>15</td>
<td>6.44·10^7</td>
</tr>
<tr>
<td>20</td>
<td>2.80·10^12</td>
</tr>
<tr>
<td>25</td>
<td>1.14·10^17</td>
</tr>
<tr>
<td>30</td>
<td>4.49·10^21</td>
</tr>
</tbody>
</table>

6.2.4 Spatial Diversity

Different from path diversity exploitation, the spatial diversity in the MuD makes the number of operations grows almost linearly with the number of receive antennas. Table 10 shows the number of operations for different number of receive antennas $Q = 1, 2$ and $3$, with $K = 15$ and $E_b/N_0 = 12$ dB.

Table 10: Number of operations for system in multipath Rayleigh channel for $K$ varying, with $E_b/N_0 = 12$ dB and BPSK modulation.

<table>
<thead>
<tr>
<th>$K$ [users]</th>
<th>PSO-MuD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mult. [$\times 10^8$]</td>
</tr>
<tr>
<td>5</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>6.77·10^2</td>
</tr>
<tr>
<td>15</td>
<td>3.27·10^7</td>
</tr>
<tr>
<td>20</td>
<td>1.42·10^12</td>
</tr>
<tr>
<td>25</td>
<td>4.49·10^21</td>
</tr>
</tbody>
</table>

6.2.5 Modulation Order

When a high-order modulation is considered, the number of operations become meaningful, since each symbol detection involves $mI$ bits per user per path, if considering to avoid handling complex-valued decision variables, as described by (15).
Table 10: Number of operations as a function of increasing receive antennas. System under flat Rayleigh channel, $E_b/N_0 = 12$ dB, $K = 15$ and BPSK modulation.

<table>
<thead>
<tr>
<th>$Q$ [antennas]</th>
<th>PSO-MuD</th>
<th>OMuD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>1</td>
<td>1.90</td>
<td>1.84</td>
</tr>
<tr>
<td>2</td>
<td>3.50</td>
<td>3.39</td>
</tr>
</tbody>
</table>

As expected, the behavior observed here is that the complexity grows significantly with the modulation order. The PSO-MuD in systems with QPSK and 16-QAM modulation still presents a manageable complexity, while the OMuD results in prohibitive implementation for any of these modulations formats under realistic system and channel scenarios. Table 11 shows the number of operations when the $E_b/N_0$ is varying, with $K = 15$ dB, QPSK modulation, $Q = 1$ antenna and $D = 1$ finger.

Table 11: Number of operations for a system under flat Rayleigh channel, $E_b/N_0$ varying, $K = 15$ and QPSK modulation.

<table>
<thead>
<tr>
<th>$K$ [users]</th>
<th>PSO-MuD</th>
<th>OMuD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>10</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>15</td>
<td>2.08</td>
<td>2.08</td>
</tr>
<tr>
<td>20</td>
<td>7.28</td>
<td>7.28</td>
</tr>
<tr>
<td>25</td>
<td>19.65</td>
<td>19.64</td>
</tr>
</tbody>
</table>

7 CONCLUSION

Under multipath channels, low and high-order modulations formats and antenna deversity, the PSO algorithm shows to be efficient for SISO/SIMO MuD asynchronous DS-CDMA problem, even under the increasing of number of multipath, system loading (MAI), NFR and/or SNR. Under a variety of analysed realistic scenarios, the performance achieved by PSO-MuD always was near-optimal but with much lower complexity when compared to the OMuD.

Errors in channel estimation deteriorate the PSO-MuD the performance, but this is inherent to the cost function calculation. Despite this drawback, heuristic MuD still remain superior with regard to conventional receiver, even with module and phase channel coefficient errors about 25% the PSO-MuD keeps much more efficient than conventional receiver with perfect channel estimation. In all evaluated system conditions, PSO-MuD resulted in small degradation performance if those errors are confined to 10% of the actual instantaneous values. Especially, for a system under medium loading, a marginal performance degradation was observed when $\epsilon_{c,\theta} < 0.05$.

Under realistic and practical systems conditions, the PSO-MuD results in much less computational complexity than OMuD. The PSO-MuD, when compared to the CD receiver, is able to reach a much better performance.

Table 12: Number of operations for $K$ varying. System under flat Rayleigh channel, $E_b/N_0 = 30$ dB and 16-QAM modulation.

<table>
<thead>
<tr>
<th>$K$ [users]</th>
<th>PSO-MuD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.04</td>
</tr>
<tr>
<td>10</td>
<td>0.63</td>
</tr>
<tr>
<td>15</td>
<td>2.08</td>
</tr>
<tr>
<td>20</td>
<td>7.28</td>
</tr>
<tr>
<td>25</td>
<td>19.65</td>
</tr>
</tbody>
</table>
with a tangible and affordable computational complexity. This manageable
complexity, of course, depends on the hardware resources available at the
receiver side (feasible at base-station), and the system requirements such as
minimum throughput, maximum admitted symbol-error-rate, and so on.

Besides, the PSO-MuD DS-CDMA accomplishes a flexible performance ×
complexity trade-off solutions, showing to be appropriate for low and high-
order modulation formats, (a)synchronous, as well as AWGN, multipath and
spatial diversity scenarios.

The increasing system loading slightly deteriorates the performance of the
PSO-MuD, but only under higher-order modulation. In all other scenarios,
PSO-MuD presents complete robustness against MAI increasing.

Finally, the complexity reduction in relation to OMuD is huge. Particu-
larly when compared with conventional receiver the complexity increment is
manageable at base-station, allowing the PSO-MuD implementation in the
third and fourth generations of mobile communication systems.

Appendix

A. Minimal Number of Trials and Single-User Performance

The minimal number of trials (TR) evaluated in the each simulated point
(SNR) was obtained based on the single-user bound (SuB) performance. Con-
sidering a confidence interval, and admitting that a non-spreading and a
spreading systems have the same equivalent bandwidth \( BW \approx \frac{1}{T_c} = BW_{\text{spread}} \approx \frac{N}{T_c} \), and thus, equivalently, both systems have the same channel response (de-
lay spread, diversity order and so on), the SuB performance in both systems
will be equivalent. So, the average symbol error rate for a single-user under
\( M \)-QAM DS-CDMA system and \( L \) Rayleigh fading path channels with exponential power-delay profile and maximum ratio combining reception is found

\[
\text{SER}_{\text{SuB}} = 2\alpha \sum_{\ell=1}^{L} p_{\ell} (1 - \beta_{\ell}) + \alpha^2 \left[ \frac{4}{\pi} \sum_{\ell=1}^{L} p_{\ell} \beta_{\ell} \times \tan^{-1} \left( \frac{1}{\beta_{\ell}} \right) - \sum_{\ell=1}^{L} p_{\ell} \right]
\]

where:

\[
p_{\ell} = \left( \prod_{k=1,k \neq \ell}^{L} \left( 1 - \frac{\nu_{k}}{\nu_{\ell}} \right) \right)^{-1}, \quad \alpha = \left( 1 - \frac{1}{\sqrt{M}} \right),
\]

\[
\beta_{\ell} = \sqrt{\frac{\nu_{\ell} g_{\text{QAM}}}{1 + \nu_{\ell} g_{\text{QAM}}}}, \quad g_{\text{QAM}} = \frac{3}{2(M-1)}.
\]

\( \nu_{\ell} = \nu_{\ell}^\text{avg} \log_2 M = m\nu_{\ell}^\text{avg} \) denotes the average received signal-noise ratio per symbol for the \( \ell \)th path, with \( \nu_{\ell}^\text{avg} \) being the correspondent SNR per bit per path.

Once the lower bound is defined, the minimal number of trials can be
defined as

\[
TR = \frac{n_{\text{errors}}}{\text{SER}_{\text{SuB}}},
\]

where the higher \( n_{\text{errors}} \) value, the more reliable will be the estimate of the
SER obtained in MCS [49]. In this work, the minimum adopted \( n_{\text{errors}} = 100, \) and considering a reliable interval of 95%, it is assured that the estimate
\( \hat{\text{SER}} \subset [0.823; 1.215] \text{SER} \). Simulations were carried out using MATLAB v.7.3
plataform, The MathWorks, Inc.

B. Monte Carlo Simulation Setup

A simplified diagram of the adopted Monte Carlo simulation setup is shown
in Figure 31. In the random data generator block, the transmitted data were
randomly generated and all information symbols had equal probability of being
selected. The transmitter block implements Equation (1), with the set of input
variable \( (A, I, K, N, M \text{ and spreading sequence type}) \) adjusted according to the
choices stated in Section 5.

The channel block adds other stochastic characteristic to the model. Here
the complex additive white Gaussian noise, with bilateral power spectral den-
sity equal to $N_0/2$, corrupts the received signal of all users. The power-delay profile for the Rayleigh fading channel can be adjusted by each user, admitting random delay and normalized power: $\sum_{\ell=1}^{L} \mathbb{E}[\gamma_\ell^2] = 1$. Then, the average SNR at the receiver input is given by: $\bar{E} = \sum_{\ell=1}^{L} \bar{E}_\ell$, where $\bar{E}_\ell = \text{SNR} \cdot \mathbb{E}[\gamma_\ell^2]$. For the exponential profile with 2 paths adopted in Section 5, $\mathbb{E}[\gamma_{1}^2] = 0.8320$ and $\mathbb{E}[\gamma_{2}^2] = 0.1680$ were assumed, with the respective delays uniformly distributed on the interval $\tau_{k,\ell} \in [0; N - 1]$.

The symbols estimated in the conventional receiver stage are used as start point for the heuristic MuDs.

The numerical results are obtained using the software Matlab\textsuperscript{R}. In order to better organize the simulation scenarios, a graphical user interface (GUI) was created, as shown in Figure 32.

**References**


[8] L. D. Oliveira, F. Ciriaco, T. Abrão, and P. J. E. Jeszensky. Particle swarm and quantum particle swarm optimization applied to ds/cdma multiuser detection...
REFERENCES


