Hybrid Local Search Polynomial-Expanded Linear Multiuser Detector for SIMO DS/CDMA Systems

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Abstract—In this work, low-complexity suboptimum detectors have been analyzed for a single-input multiple-output (SIMO) direct-sequence code division multiple access (DS/CDMA) system. As a contribution, a hybrid multiuser detector based on polynomial expansion (PE-MuD) with alpha-estimation determined by Gerschgorin circles Theorem (GCT), and followed by a low-complexity local search procedure is proposed for synchronous SIMO DS/CDMA systems. The computational complexity of the PE-MuD and the LS-MuD are expressed in terms of the total number of floating point operations in order to converge. Herein, the spatial diversity is exploited in order to improve the performance of the low-complexity multiuser detectors.

Keywords—Near-optimum search algorithms, polynomial-expanded multiuser detection, Gerschgorin circles, SIMO DS/CDMA.

I. INTRODUCTION

In a multipath channel communication environment, spatial diversity can be obtained by employment of multiple antennas at the receiver, provided that the antennas are sufficiently far apart that the channel gains between different antenna pairs can be assumed to be independent [1]. This work considers the antenna-diversity-aided detection in synchronous SIMO system and investigates the trade-off between performance gain and the complexity increasing provided by the spatial diversity scheme.

Multiuser detection algorithms usually have very high computational complexity, which greatly limits their adoption. The Decorrelator and the MMSE detectors utilize the inverse cross-correlation matrix of signature waveforms of the active users in the system (R^{−1}) to decouple the desired user’s signal and have computational complexity of O(K^3). However a multiple stage detection, presented in [2], approximately implements the inverse cross-correlation matrix through polynomial expansion in R. This scheme is namely polynomial-expanded multiuser detector (PE-MuD) and can be viewed as an iterative approach in order to approximate the linear multiuser detectors with low complexity quadratic order dependence regarding the number of users, O(K^2).

Other concept widely adopted in this current study is the local search (LS) based on neighborhood with signal detection application. The LS detection method constitutes an optimization mechanism which implement low-complexity local search solutions into a previously established neighborhood [3], [4]. The main advantage of this method lies on its inexpensive reduced complexity.

A structure formed by the PE-based detector as the first stage followed by a local search algorithm has been presented in [5]. This structure is able to offer performance improvements under DSP implementable low-complexity perspective.

In a same perspective, [6] and [7] investigate a new hybrid PE-based local search algorithm applicable to single-input single-output (SISO) DS/CDMA systems, which maintains the same convergence shape but with a smaller quantity of operations at the expense of a marginal and acceptable increasing in the bit error rate (BER). The present work considers the hybrid PE-LS-MuD in the multiple antennas environment.

II. SYSTEM MODEL

Herein, a discrete-time baseband system model is adopted, with transmission through a synchronous uplink single-input multiple-output channel, i.e., a single antenna at the mobile terminal (MT) transmitters and N antennas at the base-station (BS) receiver side, subjected to additive white Gaussian noise (AWGN) and flat Rayleigh fading. The same channel is simultaneously shared by K users, which operate under a synchronous DS/CDMA system with binary phase shift keying (BPSK) modulation. This is equivalent to a K × N MIMO system. In the transmission, the i-th information bit generated by the k-th user, at a ratio of R_b = 1/T_b bits per second is denoted by b_k, i ∈ {±1}, i = 1, 2, . . . At each i bit interval, b_k, i is modulated by a spreading sequence with pseudo-noise (PN) distribution and length L. The spreading code can be represented by the vector:

s_k, i = [s_k,1, i , s_k,2, i , . . . , s_k,L, i ]^T,

where s_k, i ∈ {±1/ \sqrt{L}} and herein L denoting the system’s processing gain, i.e., the ratio between the bit information period and the chip period, L = \frac{T_b}{T_c} = \frac{R_b}{R_c}, with R_b and R_c been the bit and chip rate, respectively. (\cdot)^T denotes the matrix transposing operator.

Take into account a scheme of detection that explores the diversity gain, which is given by the utilization of N multiple antennas at the base station, the L × 1 received signal vector at the i-th bit interval and the i-th receive antenna is:

r_{n, i} = \sum_{k=1}^{K} s_k, i c_{n,k, i} A_k b_k, i + w_n, i ,

where A_k is the amplitude of the signal transmitted by the k-th user, admitted constant across the entire message; w_n, i is the complex AWGN vector of the i-th antenna, with mean zero, variance \sigma_w^2 = N_0 and bilateral power spectral density given by N_0/2 W/Hz.

The term c_{n,k, i} denotes the complex coefficient of the channel inherent to the k-th user, at the i-th bit interval, which corresponds to received signal at the i-th antenna and is perfectly known by the receiver, but not at the transmitter side. In statistical terms, c_{n,k, i} may be represented by a circularly symmetric complex Gaussian random variable, with mean zero and variance \sigma_c^2, in the form CN(0, \sigma_c^2). In the polar form, the channel’s complex coefficient is described by:

c_{n,k, i} = |c_{n,k, i}| e^{j\theta_{n,k, i}},
where phase $\theta_{n,k}[i]$ is uniform over the range $[0, 2\pi]$, i.e., omnidirectional BS receive antenna, and independent of the magnitude $|c_{n,k}[i]|$, whose probability density function is given by Rayleigh, $f(r) = \frac{r}{\sigma_r^2}e^{-r^2/2\sigma_r^2}$, $r \geq 0$.

In the notation of matrices, with bold capital letters representing matrices and bold lower case letters representing vectors, and suppressing the bit interval index $i$ for the sake of convenience, Eq. (2) could be rewritten as follows:

$$r_n = S C_n A_b + w_n,$$  \hspace{1cm} (4)

with $A = \text{diag}(A_1, A_2, \ldots, A_K)$ being the diagonal matrix of the amplitudes of the transmitted signals, $S = [s_1, s_2, \ldots, s_K]$ being the $L \times K$ spreading code matrix, and $b = [b_1, b_2, \ldots, b_K]^T$ the bit information vector transmitted by the $K$ users. $C_n = \text{diag}(c_n)$ is the complex channel coefficients matrix, where $c_n = [c_{n,1}, c_{n,2}, \ldots, c_{n,K}]^T$ in which $c_{n,1} = e^{j\theta_{n,k}1}$, $c_{n,2} = e^{j\theta_{n,k}2}$, $\ldots$, $c_{n,K} = e^{j\theta_{n,k}K}$. Vector $w_n = [w_{n,1}, w_{n,2}, \ldots, w_{n,L}]^T$ represents the complex noise with distribution $N(0, \sigma_n^2)$. Conventional matched filters bank (MFB), also called the MFB, represents matrices and bold lower case letters representing vectors, and $\text{sgn}$ denoting the real part operator. As a result, the estimated amplitudes of the received users’ signals at the $K$th antenna, which is estimated after the application of the MRC, is described by $\hat{y}_{n,k}^\text{MRC} = \frac{y_{n,k}^\text{MFB} A_b + w_n}{M_{n,k}}$, where $M_{n,k}$ is estimated through the MRC of the received signals in phase at all $N$ antennas.

$A. \text{ Linear Methods of Multiuser Detection}$

The linear Decorrelator detector operates from multiplication of the discrete signals at the matched filters output by the inverse cross-correlation matrix $R^{-1}$. Considering the coherent reception model, the information bits vector at the $n$th antenna, which is estimated after the application of the Decorrelator filter, can be conveniently described as:

$$\hat{z}_{n,k}^\text{DEC} = \frac{y_{n,k}^\text{MFB} A_b + w_n}{M_{n,k}}$$  \hspace{1cm} (5)

with $\text{sgn}$ denoting the component-wise vector multiplication in a same way as Hadamard matrix product and $(\cdot)^*$ denoting the conjugate operator. Considering the maximum rate combining (MRC) rule applied to the received signals from the $K$ antennas, the decision variable utilized in the estimation of the information bit transmitted by the $k$-th user is given by:

$$z_{n,k}^\text{MFB} = RC_n A_b \circ p_n^* + w_n \circ p_n^*,$$  \hspace{1cm} (6)

with $\circ$ denoting the component-wise vector multiplication and $(\cdot)^*$ denoting the conjugate operator. Considering the maximum rate combining (MRC) rule applied to the received signals from the $K$ antennas, the decision variable utilized in the estimation of the information bit transmitted by the $k$-th user is given by:

$$z_{n,k}^\text{MFB} = (S^T S)^{-1} \cdot \hat{y}_{n,k}^\text{MFB}$$  \hspace{1cm} (7)

with $\text{sgn}$ representing the real part operator. As a result, the estimated information bits vector is obtained as $\hat{b}_k^\text{MFB} = (\hat{b}_1^\text{MFB}, \hat{b}_2^\text{MFB}, \ldots, \hat{b}_K^\text{MFB})^T$.

However, as well known, the performance of the MFB becomes remarkably poor when the system loading $L = K/L$ increases, i.e., due to the MAI level increasing as a function of the number of active users.
Therefore, the estimated information bit for the kth user at the output of the linear SIMO BPSK MMSE detector is given by:

\[
\hat{b}_{k}^{\text{MMSE}} = \text{sgn} \left( \Re \{ z_{n,k}^{\text{MMSE}} \} \right)
\]

(16)

III. POLYNOMIAL-EXPANDED MULTIUSER DETECTORS

The computational complexity of the linear MuDs, which originates in the operations associated to the cross-correlation matrix inversion, grows with the third order of the matrix size, i.e., \( \mathcal{O} ((mPK)^3) \), where \( P \) is the transmitted message length and \( m \) is the modulation order. However, any linear transformation matrix, represented by \( T_n \) in the SIMO model context, can be approximated through the iterative polynomial expansion method with complexity of \( \mathcal{O} ((mPK)^2) \). In general, PE methods approach approximates the cross-correlation matrix inversion via Neumann iterative series expansion.

A. General Result for PE Matrix Approximation

The general result for the \( K \times K \) PE-transformation matrix \( T_{PE} \) which is able to implement a PE multiuser detector (by approximating a matrix inversion), is given by [8]:

\[
T_{PE} = \sum_{i=0}^{N_t} w_i Q^i
\]

(17)

where \( N_t \) denotes the number of terms of the polynomial expansion. The matrix \( Q \) and the weights \( w_i \), interpreted as the coefficients for the series convergence rate, have to be chosen such that they suitably approximate the desired multiuser detector.

As a result, the polynomial expansion transformation matrix \( T_{PE} \to Q^{-1} \) when the number of expansion terms \( N_t \to \infty \).

B. Polynomial Expansion via Neumann Series

By using the Neumann series expansion method [11], the inverse cross-correlation matrix \( R^{-1} \), for the case of Decorrelator filter, can be approximated as:

\[
R^{-1} \approx T_{DEC}^{-1} = \alpha \sum_{i=0}^{N_t} [I_K - \alpha R]^{-i}, \quad \|I_K - \alpha R\| < 1
\]

(18)

where \( I_K \) is an identity matrix of size \( K \), and the associated residual error matrix is given by:

\[
e_{PE}^{\text{DEC}} = \alpha \sum_{i=N_t+1}^{\infty} [I_K - \alpha R]^{-i},
\]

(19)

such that the equality \( R^{-1} = T_{DEC}^{-1} + e_{PE}^{\text{DEC}} \) holds.

In the same way, the PE transformation matrix for the SIMO linear MMSE detector at the nth receive antenna can be approximated in:

\[
T_{PE,n}^{\text{MMSE}} \approx \left( R + \sigma_n^2 G_n^{-2} \right)^{-1} = \alpha_n \sum_{i=0}^{N_t} [I_K - \alpha_n \left( R + \sigma_n^2 G_n^{-2} \right)]^{-i}
\]

(20)

The decision variable utilized by the polynomial-expanded multiuser detector in the estimation of the information bit transmitted by the \( k \)-th user is given by using the MRC:

\[
e_{PE,n,k} = \sum_{n=1}^{N} z_{n,k}^{\text{PE}} \cdot |c_{n,k}|
\]

(21)

with \( z_{n,k}^{\text{PE}} = \{ z_{n,1}, z_{n,2}, \ldots, z_{n,K} \}^T = T_{PE,n}^{\text{DEC}} y_n^{\text{MFB}} \odot p_n^* \) for the Decorrelator approximation or \( z_{n,k}^{\text{PE}} = T_{PE,n}^{\text{MMSE}} y_n^{\text{MFB}} \odot p_n^* \) for the MMSE approximation.

Finally, the hard decisions for the PE-MuD SIMO DS/CDMA with BPSK modulation are obtained as:

\[
\hat{b}_{k}^{\text{PE}} = \text{sgn} \left( \Re \{ z_{n,k}^{\text{PE}} \} \right)
\]

(22)

C. Optimum Value of the Parameter \( \alpha \)

Since the convergence factor of an iterative method can be associated with the radius of the matricial operator, the convergence ratio is related to the dimension of this radius. The optimum parameter for linear Decorrelator detector in the PE approximation is given by:

\[
\alpha_{\text{opt}}^{\text{DEC}} = 2(\lambda_{\text{min}} + \lambda_{\text{max}})^{-1}.
\]

(23)

In turn, for the linear MMSE detector approximation, the optimum value of \( \alpha \) is given by:

\[
\alpha_{\text{opt}}^{\text{MMSE}} = 2(\lambda_{\text{min}} + \lambda_{\text{max}} + 2\sigma_n^2)^{-1}.
\]

(24)

Important to point out that the deterministic choice of \( \alpha_{\text{opt}} \) through cross-correlation matrix eigenvalues calculation is prohibitively complex for the implementation of the polynomial expansion method using practical digital signal processing hardware platforms. As a result, the complexity of only one eigenvalue computation, as well as of all eigenvalues calculation from a \( K \) squared-dimension matrix results in \( \mathcal{O} (K^3) \). Thus, it is necessary to estimate the optimum value of the parameter \( \alpha \). Next, the estimation of \( \alpha_{\text{opt}} \) is suggested by using the Gerschgorin circles Theorem [11].

D. Gerschgorin Circles Theorem (GCT)

According to Gerschgorin Theorem, any eigenvalue \( \lambda_i \) of a matrix \( R \), which has elements \( r_{ij}, \forall i,j \), is situated in one of the circles of the complex plane that are centered in \( r_{ii} \):

\[
|\lambda_i - r_{ii}| \leq \sum_{j \neq i} |r_{ij}|.
\]

(25)

Thus, through a simple calculation, by using the elements of \( R \), the approximated values of \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \), which are denoted by \( \hat{\lambda}_{\text{min}} \) and \( \hat{\lambda}_{\text{max}} \), respectively, can be achieved by:

\[
\hat{\lambda}_{\text{min}} \approx \min \left\{ r_{ii} + \sum_{j \neq i} |r_{ij}| \right\}, \quad \forall i,
\]

(26)

\[
\hat{\lambda}_{\max} \approx \max \left\{ r_{ii} + \sum_{j \neq i} |r_{ij}| \right\}, \quad \forall i.
\]

(27)

The GCT guarantees a considerable reduction in the complexity of the calculation of the minimum and maximum eigenvalues; therefore, GCT results is applied directly to Eq. (23) and (24) in order to estimate the parameter \( \alpha \).

IV. LOCAL SEARCH METHODS APPLIED TO MUD

The well known local search methods propagate the attainment of near-optimum solutions from searches guided in subspaces of the dimension of the optimization problem. The lopt-LS-MuD algorithm performs guided searches for the b vector that maximizes the linearly combining cost function:

\[
f(b) = \frac{1}{N} \sum_{n=1}^{N} \Omega_n(b),
\]

(28)

where \( \Omega_n \) is the Euclidean distance function between the received signal at nth antenna and the reconstructed signal at the receiver from the information candidate-vector, \( b \):

\[
\Omega_n(b) = 2R \left\{ b^T C_n^m A^m_n \right\} - b^T C_n AR A^m_n b.
\]

(29)
The 1opt-LS-MuD algorithm selects candidate-vectors with unitary Hamming distance\(^1\) from the MFB output vector. Details related to this LS algorithm can be found in [4], [12].

 herein, the users’ power profile is considered for classification purpose at the beginning of the guided search process. From the estimated users’ received amplitudes diagonal matrix \( G_n = M_n A = \text{diag}(g_{n,1}, g_{n,2}, \ldots, g_{n,K}) \), the average received amplitude of the \( k \)th user can be estimated as a linear combination of the corresponding \( N \) antenna branches:

\[
g_k = \frac{1}{N} \sum_{n=1}^{N} g_{n,k}, \tag{30}
\]

where vector \( g = [g_1, g_2, \ldots, g_K]^T \) is formed to assist the users’ power classification. In the following, an adaptation for the 1-opt LS algorithm is proposed and a new algorithm is formed.

A. Local Search Algorithm 1-adapt LS

The quantity of calculations of the cost functions during the search for the best candidate vector can be limited by using a given threshold. Chase establishes a threshold criterion based on channel measurement informations, by selecting a fixed number of the lowest confidence bits to be changed [13]. Differently of Chase search stop criterion, herein for the proposed 1-opt LS algorithm, a dynamic threshold is used in order to create adaptation and reduce complexity. The algorithm one-adaptive local search (1-adapt LS) classifies the received signals in order of increasing amplitude; then, candidate vectors with unitary Hamming distance are generated, following the ordering of the signals (from the weakest to the strongest), and their respective cost functions are evaluated; in case of the linearly combining cost function value does not increase inside of a pre-established quantity of consecutive evaluations, denoted here by parameter \( \kappa \), the search process is interrupted and a new search is initiated. In general, \( \kappa \) is taken as a fraction of the number of active users in the system. The pseudo-code for the algorithm 1-adapt LS is described in the Algorithm 1.

B. Hybrid PE-MMSE 1opt-LS-MuD and 1adapt-LS-MuD

A detection structure formed by a suboptimal local search algorithm in conjunction with a primary stage of polynomial-expanded linear multiuser detector was presented in [5]. This structure has been reproduced herein, by deploying in the first stage, the SIMO polynomial-expanded MMSE detector with \( \alpha \) estimated via the Gerschgorin circles method, and in the second stage, the 1-opt LS algorithm. In this work, the low-complexity 1-adapt LS algorithm is deployed in the second stage of the hybrid detection structure to form the hybrid 1adapt-LS-MuD with spatial diversity exploration. In Section VI a performance comparison including both hybrid PE-LS-based suboptimal MuDs have been carried out.

V. COMPUTATIONAL COMPLEXITY

The metric of computational complexity is defined by the total number of floating points operations (FLOP) needed for each detector to achieve convergence. The considered operations are: multiplication, comparison, random number generation and selections. The complexity is expressed as a function of the number of users (\( K \)), antennas (\( N \)), iterations needed for convergence (\( n_{it} \leq N_{it} \)) and the average quantity of cost function calculations by iteration, which is denoted by \( \zeta_{avg} \). The cost function calculation in (29) is the most significant factor in determining the complexity of the detectors.

The terms \( 2^n M^{w_M} n \) and \( C_n A R A C_{n}^{H} \) are evaluated outside the iterations loop and adopted constant during the detector search. The resulting number of operations needed for these two terms is \( 4K^3 + 6K^2 + 2K \), and this calculation is done \( N \) times (one for each antenna). Inside the iterations loop, the number of operations needed for each candidate vector evaluation through cost function becomes \( N(2^K + 3K) \).

The Table I shows the complexity for the PE-MMSE-MuD with \( N_{it} \) terms and estimated \( \alpha \), the linear MMSE-MuD and the local search algorithms 1-opt LS and 1-adapt LS.

\[
\begin{array}{|c|c|}
\hline
\text{MuD} & \text{Operations} \\
\hline
\text{Linear-MMSE} & N \left( 2K^3 / \beta + 3K + 2 \right) (K - 1) \\
\hline
\text{PE-MMSE} & N \left( 3K^2 + \beta \right) \left( N (N_{it} - 1) + 1 \right) \\
\hline
\text{1-opt LS} & n_{it} \left( KN (K^2 + 3K + 2K) + N (4K^2 + 6K^2 + 4K) + 1 \right) \\
\hline
\text{1-adapt LS} & n_{it} \left( \zeta_{avg} (K^2 + 3K) + 3K + 3 \right) + N (4K^2 + 6K^2 + 4K + 1) \\
\hline
\end{array}
\]

VI. PERFORMANCE-COMPLEXITY ANALYSIS

The suboptimal MuD performances are evaluated by means of Monte Carlo simulation (MCS) method. The flat Rayleigh fading channels with magnitude and phase coefficients perfectly estimated at the receiver side have been assumed, while the number of antennas at receiver ranges from \( N = 2 \) to
The proposed hybrid PE-MMSE 1-adapt-LS algorithm promotes a remarkable gain in the SIMO DS/CDMA system performance equipped with polynomial expansion-based hybrid multiuser detectors. When associated to low-complexity PE-MuDs, it provides reliability to the detection process, without an excessive increasing in its implementation cost, been able to offer a promising performance-complexity trade-off.

**REFERENCES**


