Lattice Reduction-Aided MIMO Detectors under Correlated MIMO Channels

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Abstract—In this contribution, lattice reduction (LR) technique is applied to improve the MIMO detector performance under correlated channel constrains. Zero-forcing, minimum mean squared error (MMSE), ordered successive interference cancellation (OSIC) and sphere decoding (SD) detectors are analysed taking into consideration a) different correlated fading channel indexes, and b) increasing spectral efficiency, by combining number of transmit antennas and modulation formats. Analyses of correlated channel effects over the performance of MIMO systems equipped with different LR-aided detectors are carried out, indicating the robustness of those detectors, as well as the SD-MIMO detector deficiency to deal with such large number of strongly correlated MIMO channel condition. Besides, computational complexities are compared aiming to determine the best LR-MIMO detection scheme under the perspective of performance-complexity tradeoff.

Keywords—Lattice reduction; correlated channels; Large MIMO detectors; zero-forcing; MMSE; sphere decoding.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems provide a significant spectral efficiency and/or performance improvement on wireless communication systems by the use of multiple antennas at both transmitter and receiver side [1]. Here, parallel data streams are transmitted using multiple antennas to increase the spectral efficiency at the cost of increased complexity for data detection at the receiver [2]. MIMO systems achieve this goal by dividing the total transmit power over the antennas, taking advantage of the multipath diversity in order to achieve significant array gain (more bits per second per hertz of bandwidth), or alternatively to achieve a reliability (diversity gain mode) on the received information, being suitable to reduced channel fading effects.

Under large (or dense) MIMO regime, the communication systems use antenna arrays with an order of magnitude, in terms of the number of elements, bigger than in systems being built today, say tens (a hundred) antennas or more. Very large MIMO arrays is a new research field both in communication theory, propagation, and electronics and represents a paradigm shift in the way of thinking both with regards to theory, systems and implementation. A recent survey on the theoretical research on signal processing, coding and network design for very large MIMO systems is presented in [3]. The authors discuss several aspects of dense-MIMO systems, such as information-theoretic performance limits, practical algorithms, influence of channel properties on the very large MIMO system performance, as well as practical constraints on the antenna arrangements.

Linear detection techniques such as linear zero forcing (ZF), minimum mean squared error (MMSE), successive interference cancellation (SIC), and ordered SIC (OSIC) presents a performance clearly inferior to the maximum likelihood (ML) detector. The sphere decoding (SD) [4] can be considered when looking for a near-optimum performance. However SD approach results in a prohibitive complexity for implementation under low or low-medium signal-to-noise ratio (SNR) region on real communication systems, becoming of the same order of ML complexity for low SNR region [5].

The LR is a mathematical concept used to solve many problems involving lattice points. In the MIMO signal detection problem, the LR can be used to improve the conditioning of the channel matrix [2], [6], thus allowing to use simpler detectors [2], and consequently less computational complexity is necessary to maintain acceptable performance [6]. A powerful and well-known algorithm for reduction is the LLL, proposed by Lenstra, Lenstra & Lovasz in 1982 [7]. The LLL algorithm is a polynomial time algorithm that finds short vectors within an exponential approximation factor.

The contribution of this work is twofold: i) quantify the impact on the MIMO system performance improvement when lattice reduction technique is deployed to mitigate the effects of channel correlation; ii) determine the best MIMO detector choice among the popular SD and ZF, MMSE, MMSE-OSIC LR-aided MIMO detectors in terms of complexity-performance tradeoff. More precisely, in this contribution, lattice reduction technique is applied to improve the MIMO detector performance under correlated channels constrains. The MIMO detectors are analysed taking into consideration a) correlated fading channels, and b) increasing number of transmit and receive antennas. Analysis of correlated channel effects over the MIMO system performance equipped with different LR-aided detectors are carried out. Besides, computational complexities are compared aiming to determine the best LR-MIMO detection scheme under the perspective of performance-complexity tradeoff.

This work is organised as follows: system model is developed in section II, as well as a discussion related to correlated channel, and the various available MIMO detectors. Section III shows how the LR technique can be employed in order to improve the performance of these schemes. Simulation results, including complexity and performance analyses for those MIMO detectors are carried out in section IV. Final remarks and conclusions are offered in section V.
Notation: boldface lower case letters are used to denote vectors and boldface upper case letters to denote matrices. The superscripts $(\cdot)^{-1}$, $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^+$ denote the inverse, conjugate transpose, transpose and Moore-Penrose pseudo-inverse, respectively, while $I_n$, represents the identity matrix of order $n$. The notation $\mathbb{C}^{a \times b}$ refers to the space of complex matrices with $a$ rows and $b$ columns, $||x||$ denotes the Euclidean norm and $||x||_2$ the spectral matrix norm. Boldface lower case letters with tilde denotes received signal vectors, and boldface lower case with hat denotes estimated signal vectors, i.e., received signal after filter decision.

II. WIRELESS MIMO SYSTEM MODEL

We consider $n_T$ transmit antennas and $n_R$ receiver antennas, with $n_R \geq n_T$, where data is demultiplexed into $n_T$ transmit antennas. A MIMO system topology is depicted in Fig. 1.

A classical problem in MIMO system consists in reliably detect the transmitted symbol $s \in \mathbb{C}^{n_T \times 1}$, despite the channel’s distortion and noise [8]. The receive signal is given by:

$$r = Hs + n,$$  

where $r \in \mathbb{C}^{n_R \times 1}$ is the received symbol, $H \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix that is known a priori, usually, it is estimated beforehand, and $n \in \mathbb{C}^{n_R \times 1}$ is the additive Gaussian noise samples such that $n \sim \mathcal{CN}\{0; \sigma_n I\}$. Those variables are complex values. For simplicity, we can rewrite (1) as real and imaginary part separately:

$$x = \Re\{Hs + n\},$$  

where the channel matrix

$$H = \begin{bmatrix} \Re\{H\} \\ \Im\{H\} \end{bmatrix} \in \mathbb{R}^{n_R \times mn}$$

and the real-valued vectors

$$x = \begin{bmatrix} \Re\{x\} \\ \Im\{x\} \end{bmatrix}, \quad s = \begin{bmatrix} \Re\{s\} \\ \Im\{s\} \end{bmatrix}, \quad n = \begin{bmatrix} \Re\{n\} \\ \Im\{n\} \end{bmatrix}.$$  

Note that $m = 2n_T$, $n = 2n_R$, $x, n \in \mathbb{R}^{n_R \times 1}$ and $s \in \mathbb{R}^{n_R \times 1}$.

In this work we will consider high order $M$ quadrature amplitude modulation (M-QAM). The complex-valued symbol (finite) set is given by $S = \{\mathcal{A} + \sqrt{-1} \cdot \mathcal{A}\}$, where $\mathcal{A} = \{\pm \sqrt{a}; \pm \sqrt{a}; \ldots; \pm \sqrt{M-1}\}$ is the real-valued finite set. The parameter $a = \sqrt{\frac{B}{M-1}}$ is used for normalizing the power of the complex valued transmit signals to 1.

Several MIMO detectors are described in the following, and their performance compared. However, before we deal with MIMO detectors, one important and realistic aspect related to the MIMO matrix coefficients is modelled: the MIMO channel correlation effect.

A. Correlated MIMO Channels

In the last two decades, various MIMO channel correlation models have been proposed; among them, there is an important class of MIMO channel models that assume the correlation among receive antennas (Rx) is independent of the correlation between transmit antennas (Tx) (and vice versa). Hence, admitting the independent correlation hypothesis among Rx and Tx antennas, the MIMO channel model for Rayleigh flat-fading channels can be modelled as [9]:

$$H = \sqrt{R_{R,XX}} G \sqrt{R_{R,XX}}.$$  

where $G \in \mathbb{C}^{n_R \times n_T}$ is an independent, identically distributed (i.i.d.) complex Gaussian with zero-mean and unit variance elements. The correlation matrices $R_{R,XX} \in \mathbb{R}^{n_T \times n_T}$ and $R_{R,XX} \in \mathbb{R}^{n_R \times n_R}$ denote the correlation observed among the transmitter antennas and receiver antennas, respectively. Note that matrix $G$ is similar to matrix $H$.

Without loss of generality, we assume in this work that the Tx and Rx antennas are equally separated, with equal number of antennas $n_T = n_R$ and the correlation matrix $R_{R,XX} = R_{R,XX} = R_{R,XX}$. Hence, the matrix $R_{R,XX}$ can be written as:

$$R_{R,XX} = \begin{bmatrix} 1 & \rho & \rho^4 & \ldots & \rho^{(n_T-1)^2} \\ \rho & 1 & \rho & \ldots & \rho \\ \rho^4 & \rho & 1 & \ldots & \rho \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{(n_T-1)^2} & \rho & \ldots & \rho & 1 \end{bmatrix},$$  

where $\rho$ is the normalized correlation index. Note that a totally uncorrelated scenario means $\rho = 0$, while a fully correlated scenario implies $\rho = 1$. Besides, in the numerical results we have assumed also three intermediate scenarios: $\rho = 0.2$ representing a weakly correlated channel, $\rho = 0.5$ for a medianly correlated channel and $\rho = 0.9$ for a strongly correlated channel.

B. ML MIMO Detector

The optimum maximum-likelihood (ML) detector searches over the whole set of possible symbols $s \in \mathcal{A}^{n_T}$, and decides in favour of the symbol that minimises the Euclidean distance among the reconstructed $Hs$ and received $x$ signal: %vspace-1mm

$$\hat{s}_{\text{ML}} = \arg \min_{s \in \mathcal{A}^{n_T}} ||x - Hs||^2.$$  

This search complexity is exponential according to the number of antennas and number of symbols, i.e., it is of order $O(M^{n_T})$. This way, it becomes impracticable in dense MIMO systems (large number of antennas) and/or higher order modulations formats, or even under high number of symbols per time-slot jointly detected.

C. ZF MIMO Detector

Looking for suppressing the channel interference, the zero-forcing (ZF) detector has been conceived to suppress completely the interference by multiplying the received signal vector $x$ to the Moore-Penrose pseudo-inverse of the channel matrix $H^+ = (H^TH)^{-1}H^T$. The output signal from the detector is given by:
\[
\tilde{s}_{ZF} = H^+x = s + (H^TH)^{-1}H^Tn \quad (8)
\]
This output signal is mapped to the alphabet symbol over the decision step, which consists in approximate the received symbol to the closer alphabet symbol according to a threshold.

Note that, for a scenario with no noise, ZF is identical to ML, since all channel interference is suppressed. However, ZF leads to noise amplification in a noisy scenario, because \( H^+n \) is larger than \( n \) [10].

**D. MMSE MIMO Detector**

Trying to improve ZF detector performance, the minimum mean square error (MMSE) detector for MIMO system have been developed. The MMSE detector takes the noise term into account, thus the performance can be improved [10]. Hence, we define the \((n + m) \times m\) extended channel matrix \( \tilde{H} \) and the \((n + m) \times 1\) extended receive vector \( \tilde{x} \) by

\[
\tilde{H} = \begin{bmatrix} H \\ \sigma_n I_m \end{bmatrix} \quad \text{and} \quad \tilde{x} = \begin{bmatrix} x \\ 0_{m,1} \end{bmatrix}, \quad (9)
\]

where \( 0_{i,j} \) is a \( i \times j \) matrix of zeros. Then, the output signal from MIMO MMSE detector is:

\[
\tilde{s}_{MMSE} = (H^TH + \sigma_n^2I_m)^{-1}H^Tx = (H^TH)^{-1}H^Tx = H^+x. \quad (10)
\]

Note that this equation has the same structure of (8), whereas the vector signal and the channel matrix have been extended.

**E. Ordered Successive Interference Cancellation (OSIC) MIMO Detector**

Evaluating the QR decomposition interference cancellation (OSIC) MIMO detector, such that \( H = QR \), the MIMO detection can be performed to each layer successively, by taking:

\[
\tilde{s}_{OSIC} = Q^Hx = Rs + Q^Hn. \quad (11)
\]

Since \( Q \) is an unitary matrix, the statistical properties of the noise term \( Q^Hn \) remain unchanged. Due to the upper triangular structure of \( R \), the \( n \)-th element of \( \tilde{s}_{OSIC} \) is totally free of inter-antenna interference and can be used to estimate \( s_n \) after appropriate scaling by a factor \( 1/r_{n,n} \) [11]. Removing its interference from the received signal \( x \), and assuming that the estimated symbol is correct, the further symbols can be detected as if there were no previous layers, in a simpler equivalent system. However, if an error occur on the first layers, it will be propagated until the end of the algorithm, deteriorating its performance. Hence, a remarkable performance improvement can be achieved detecting the most reliable layers first. It can be done as shown in [11], by evaluating the MMSE sorted QR decomposition (SQRD) of the channel matrix, and performing the remainder of the algorithm on the same way as described above.

**F. Sphere Decoding (SD) MIMO detector**

A detector similar to ML has been developed in the last decade, namely the sphere decoding (SD) detector [4], [5]. As advantage, the SD detector results in a lower complexity\(^1\), since it does not realize an exhaustive search on the entire alphabet. SD solves (7) seeking the solution inside a hypersphere of radius \( d \):

\[
||x - Hs||^2 \leq d^2. \quad (12)
\]

This radius is determined using the pruning technique, described in [8], [12]. Moreover, in order to obtain candidate-solution points inside the hypersphere without an exhaustive search, the search is made taking advantage of the triangular structure of the \( R \) matrix, given by the QR decomposition of the channel matrix \( H \), such that \( H = QR \). Hence, the points inside the hypersphere can be found layer-by-layer, starting from the last row of \( R \), from the partial Euclidean distances, by evaluating:

\[
||Q^Tx - Rs||^2 \leq d^2 \quad (13)
\]
at that layer, and summing the previous. Then, enumerating the points inside the hypersphere, the solution is the one closest to the hypersphere centre \( Q^Tx \).

**III. LATTICE REDUCTION AIDED MIMO DETECTION**

As already mentioned, the channel matrix is not completely orthogonal, and even in some practical scenarios of interest can result in highly correlated matrices, which prejudices the detection process and mainly deteriorates the MIMO system performance. Hence, to circumvent this problem, we aim to turn the channel matrix as near-orthogonal as possible, looking for improve the MIMO detection process with a manageable complexity increasing.

In the next, our main concern is to analyse the LR-aided MIMO detectors under high number of antennas, higher order modulation, and correlated channels scenarios.

**A. LR-aided Detectors in Dense MIMO System**

Using \( \tilde{H} = HT \) and \( z = T^{-1}s \), the received signal vector in (2) can be rewritten as

\[
x = Hs + n = HTT^{-1}s + n = \tilde{H}z + n. \quad (14)
\]

Note that \( Hs \) and \( \tilde{H}z \) describe the same point in the lattice. However \( \tilde{H} = HT \) is usually much more near to the orthogonality than \( H \). The domain of \( z \) is \( T^{-1}\mathcal{A}^m \), differently of \( s \) which is \( \mathcal{A}^n \). This difference can be seen as a transformation applied to \( \mathcal{A}^m \), i.e. the lattice points are resized and translated. The \( T \) is an unimodular matrix, i.e., a square integer matrix with determinant \( \pm 1 \), obtained from LR algorithm; note that LLL algorithm [7] is adopted herein.

The idea of LR-aided detection in dense MIMO system is to consider the system model in (14) and perform the quantization on \( z \) instead of \( s \) with a large number of antennas. For LR-aided ZF the output signal is written as:

\[
\tilde{z}_{LR-ZF} = T^{-1}\tilde{s}_{ZF} = \tilde{H}^+x, \quad (15)
\]

where the multiplication by \( \tilde{H}^+ \) usually causes less noise amplification due to more orthogonal columns of the channel matrix.

Similarly, the output signal of LR-aided MMSE is:

\[
\tilde{z}_{LR-MMSE} = T^{-1}\tilde{s}_{MMSE} = \tilde{H}^+x. \quad (16)
\]

\(^1\)Specially for medium and high SNR regions.
Note that lattice reduction operations consists in scaling and shifting of the lattice points, so it is necessary to introduce re-scaling and re-shifting operations over the signal vectors before the quantization [13], [14]. In [14], it is showed that the estimate in the reduced lattice domain is performed by:

\[ \hat{\mathbf{z}} = 2 \left( \frac{\mathbf{z} - \beta' \mathbf{T}^{-1} \mathbf{1}}{2} \right) + \beta' \mathbf{T}^{-1} \mathbf{1}, \]  

where \( \mathbf{1} \) is a vector of ones, \( \beta' \) is a parameter that depends on the modulation format, being \( 1 + j \) for all \( M\)-QAM and 1 for pulse amplitude modulation (PAM) and binary phase shift keying (BPSK) modulation; \( \lfloor \cdot \rfloor \) denotes the rounding operator.

Finally, the signal vector estimation in the original alphabet is obtained applying the transformation \( \mathbf{x} = \mathbf{T} \hat{\mathbf{z}} \).

### IV. Numerical Results

In the sequel, the bit error rate (BER) versus \( E_s/N_0 \) performance analysis under perfect channel estimation, different number of antennas, modulation order, as well as different MIMO channel correlation indexes have been considered. Important to determine the best complexity \( \times \) performance, in Section IV-B the computational complexity analysis for all MIMO detectors considered in this work is carried out. Specifically, three configurations for the equal number of transmitted and received antennas, \( n_T = n_R \), modulation format \( M\)-QAM and channel correlation values \( \rho \) have been considered.

#### A. Performance Analysis

Figure 2 shows the BER performance of the considered detectors for a 64-QAM MIMO system with \( n_T = n_R = 4 \) antennas under correlated channels with coefficients perfectly estimated; accordingly, Fig. 3 and 4 depict the BER performance for \( 8 \times 8 \) 16-QAM and \( 20 \times 20 \), 4-QAM, respectively.

As one can see from Fig. 2, 3 and 4, the three LR-aided MIMO detectors achieve the full diversity degree, defined as the asymptotic slope of the BER curve in high SNR region, although its performance in low SNR region has proved worse, specifically for the larger number of antennas. The LR technique applied to the MIMO systems, specially under large MIMO condition, makes them more sensitive to noise, although being robust against the correlation between antennas, what makes its BER performance improves considerable at the high SNR region, where the additive noise is negligible. Furthermore, for strongly correlated channels under large number of antennas (tens or more), the MMSE OSIC and the LR-MMSE OSIC MIMO detectors showed a poor performance, as indicated in Fig. 3 and 4. It can be explained by the inefficiency of the sorting procedure under large number of strongly correlated MIMO channels condition, when the columns of the channel matrix become more and more similar, i.e. rank deficiency, and therefore causing propagation errors in the serial detection process more often, what explains the performance behavior in high SNR of the Fig. 3 and 4.

#### B. Complexity

According to [15], the algorithm complexity can be evaluated in terms of the total number of floating-point operations.
(flops), which one flop is defined as an addition, subtraction, multiplication or division between two floating point numbers. The complexity in terms of number of flops for the same matrix operations can be found on [15], and LLL procedure on [16]. Using this methodology, we calculate the complexity of MIMO detectors shown in Table I, where \( N = n_T = n_R \) is the number of antennas, and \( M \) is the modulation order in \( M\)-QAM. The complexity of the SQRD from MMSE-OSIC can be found in [11], while the parameters \( p_1, p_2, \) and \( p_3 \) were determined from [16]. The complexity of SD detector has been determined using results of [5].

### Table I

<table>
<thead>
<tr>
<th>Detector</th>
<th>Total Complexity</th>
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<tbody>
<tr>
<td>ML</td>
<td>( M^N (8N^2 + 2N) )</td>
</tr>
<tr>
<td>ZF</td>
<td>( 14/3N^3 + 2N^2 )</td>
</tr>
<tr>
<td>MMSE</td>
<td>( 26/3N^3 + 4N^2 )</td>
</tr>
<tr>
<td>MMSE-OSIC</td>
<td>( 40/3N^3 + 13/3N^2 + 25/6N )</td>
</tr>
<tr>
<td>LR-ZF</td>
<td>( 14/3N^3 + 2N^2 + 2N^3 + Np_1 + 2Np_2 + p_3(7N + 1) )</td>
</tr>
<tr>
<td>LR-MMSE</td>
<td>( 26/3N^3 + 4N^2 + 2N^3 + Np_1 + 2Np_2 + p_3(7N + 1) )</td>
</tr>
<tr>
<td>LR-MMSE-OSIC</td>
<td>( 40/3N^3 + 13/3N^2 + 25/6N + 2N^3 + Np_1 + 2Np_2 + p_3(7N + 1) - N^3 )</td>
</tr>
<tr>
<td>SD</td>
<td>( (N^2 + N + 1)2^{2N} )</td>
</tr>
</tbody>
</table>

The number of complex operations for all those considered MIMO detectors according to the number of antennas and modulation order has been drawn in Fig. 5. As mentioned before, the ML and SD MIMO detectors present an exponential complexity regarding \( M \) and \( N \). The ML-MIMO detector requires a huge number of operations, what makes it not feasible even for a low number of antennas. The complexity of SD-MIMO detector depends primarily on the SNR: under low SNRs, the SD complexity increases faster (regarding the problem dimension, \( M \) and \( N \)) than under high SNRs regime. Besides, the SD complexity is also sensitive to the MIMO correlation channel, aggregating a progressive complexity overhead when channel correlation index \( \rho \geq 0.5 \), what justifies the fact that under high correlation MIMO channels and number of antennas (Fig. 4.c), the SD algorithm is not able to obtain candidate-solution points inside the hyper-sphere, i.e. it is not able to achieve suitable BER performance.

### V. CONCLUSIONS

Lattice reduction (LR) technique has been applied to improve the MIMO detectors performance under correlated channel. Analysis of correlated channel effects over the MIMO system performance equipped with different LR-aided detectors has indicated its robustness. Besides, among the analysed detectors, the LR-MMSE MIMO OSIC achieves the best performance-complexity tradeoff. Furthermore, we have found that the SD-MIMO detector is very sensitive to correlated channel condition, increasing remarkably its complexity when the MIMO channel becomes strongly correlated (\( \rho >> 0.5 \)).

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**REFERENCES**


