Energy Efficiency Design in MC-CDMA Cooperative Networks

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Abstract—The energy maximization in MC-CDMA cooperative wireless networks is a NP-hard optimization problem of great interest in future networks systems as well as wireless sensor networks. This paper presents a game theoretic approach for energy efficiency (EE) maximization in multi-carrier code division multiple access (MC-CDMA) wireless cooperative networks considering receiver single-user or multiuser design, as well as distributed implementation approach in order to solve the EE design problem. Moreover, the iterative water-filling algorithm (IWFA) and the Verhulst distributed power control algorithm (V-DPCA) are employed to solve the inner loop of the proposed EE maximization algorithm. A study over the quasi-concavity of the utility function is presented while numerical results are offered to corroborate the mathematical model, as well as to verify the better performance of the IWFA over the V-DPCA. Indeed, the superiority of the IWFA over V-DPCA in terms of both EE and SE is evident. This may be explained through the fact that the IWFA optimizes the power allocation of each user through all sub-carriers at the same time, while Verhulst-based DPCA performs the power control on each sub-channel considering all users.

Index Terms—Cooperative Networks; MC-CDMA; Energy Efficiency design; iterative water-filling; multiuser detection; Verhulst distributed power control algorithm.

I. INTRODUCTION

Resource allocation in wireless multiple access networks has been the focus of many studies over the last decades due to two main scarce resources present in these systems: power and spectrum. The first one is clearly limited due to the battery capacity of mobile terminals and the second one is a natural limited resource that has suffered even more limitation thanks to the creation of numerous new services such as live streaming, social networks, cloud storage and so on.

A. Motivation

The study of resource allocation in wireless networks is an important topic because it impacts in companies profits and users satisfaction. These two factors are also influenced by current technologies inability to provide more bandwidth at low operational costs. Thus, proposing a easily deployable resource allocation algorithm is highly desirable in order to increase both energy and spectral efficiencies.

B. Related Work

Many studies have been conducted recently aiming to find a good resource allocation algorithm in cooperative networks. In this context some works may be highlighted [1]–[6]:

In [1] a DPCA for direct sequence CDMA (DS/CDMA) single-rate networks based on Verhulst equilibrium is proposed, and then extended in [2] to multi-rate scenario, while in [3] a game theoretic approach using IWFA is proposed in order to solve the power control problem in MC-CDMA networks. In [4] a energy-efficient approach for power control and receiver design in wireless networks is presented. [5] presents a game theoretic approach for power control and receiver design in cooperative DS/CDMA networks. Finally, in [6] an analysis over EE and SE tradeoff in DS/CDMA networks considering receiver design is presented.

C. Organization

This paper is organized as follows: Section II presents the system model and description; energy and spectral efficiencies as well as problem formulation is presented in Section III. Furthermore, Section IV presents the game theoretic approach and the algorithms used to solve the EE maximization problem. Numerical examples and results are offered in Section V. Finally, conclusions are given in Section VI.

II. SYSTEM DESCRIPTION

In a single rate uplink multi-carrier code division multiple access system the equivalent base-band signal of each sub-carrier $k$ received at a fixed relay station (FRS) can be mathematically described as:

$$y(k) = \sum_{i=1}^{U} \sqrt{p_i(k)}h_i(k)b_i(k)s_i + \eta_k,$$  
(1)

where $U$ is the system loading, $h_i$ is the complex channel gain between the $i$th user and the relay station, assumed constant during the entire symbol period, $s_i$ is the $i$th user spreading code with length $F_i$ and is defined as $s_i \triangleq \frac{1}{\sqrt{c_F}} [c_1, c_2, \ldots, c_{F_i}]$ with $c_i = U[-1, 1]$, representing the processing gain; the modulated symbol is given by $b_i$, and $\eta_k$ is the relay thermal noise vector, assumed to be additive white gaussian noise (AWGN), zero-mean and covariance matrix given by $\sigma^2 I_N$.

The uplink $U \times 1$ channel gain vector, considering path loss, shadowing and fading effects, between user $i$ and the relay at the $k$-th sub-channel is given by:

$$h(k) = [h_1(k) \ h_2(k) \ \ldots \ h_U(k)]^\top \quad k = 1, \ldots, N$$  
(2)

which could be assumed static or even dynamically changing over the optimization window; $N$ is the number of total non-overlapped subcarriers available and $(\cdot)^\top$ is the transpose operator. Besides, we can define the channel gain coefficient between the single fixed relay station (FRS) and the destination (BS) considering path loss, shadowing and flat fading effects at the $k$th subcarrier as $g(k)$; hence, the $N \times 1$ relay-destination channel gain vector can be defined accordingly:

$$g = [g(1) \ g(2) \ \ldots \ g(k) \ \ldots \ g(N)]$$  
(3)

Fig. 1 illustrates the MTs, FRS and BS macrocell positioning scenario.

1Mobile channel is assumed to be slow and non-selective in frequency.
According to the system configuration, sketched in Fig. 1, due to large distance and natural obstacles between mobile terminals (MTs) and the base station (BS), the direct path between MT-BS is despaired, i.e. all mobile terminals must communicate with the relay-station in order to transmit. Thus, the equivalent base-band received signal at the FRS is first normalized by \( \sqrt{\text{PN}(k)} = \sqrt{\mathbb{E}[|y(k)|^2]} \). Considering the noise and information symbols from each user uncorrelated, the normalized received power is expressed by

\[
P_N(k) = \sum_{i=1}^{U} p_i(k) h_i(k) + \sigma^2 F_r(k)
\]

where \( h_i(k) = [h_i(k)]^2 \) is the channel power gain between user and relay. Afterwards, the received vector \( y(k) \) is amplified by the \( U \times U \) matrix \( \mathbf{A} \) constrained by \( \text{tr}(\mathbf{A} \mathbf{A}^H) \leq p_r \) where \( p_r \) is the available power at the FRS. Therefore, the signal obtained at the base-station is:

\[
y(k) = \frac{g(k)}{\sqrt{\text{PN}(k)}} \left( \sum_{i=1}^{U} \sqrt{p_i(k)} h_i(k) b_i(k) \mathbf{A} s_i(k) + \mathbf{N}_b + \eta_{bs} \right)
\]

where \( \eta_{bs} \) is the base station thermal noise, assumed AWGN with covariance matrix \( \sigma^2 I \).

In multiple rates scenarios each user can modify its processing gain \( F_r(k) > 1 \) at each sub-carrier in order to satisfy different data rates, such that \( F_r(k) \) is defined as:

\[
F_r(k) = \frac{r_c}{r_i(k)} = \frac{\mathbb{W}}{r_i(k)}
\]

where \( r_c \approx \mathbb{W} \), with \( \mathbb{W} \) being the system bandwidth, and \( r_i(k) \) the data rate for the \( i \)th user at the \( k \)th sub-carrier.

In multiple access interference-limited networks an important QoS measure is the signal to interference plus noise ratio (SINR) since all users transmit over the same channel at the same time causing what is known as multiple access interference (MAI), which is responsible for the soft-capacity of CDMA systems. Hence, the total available spectrum may be divided in to \( N \) uncorrelated CDMA non-selective sub-channels in order to improve capacity and system throughput, this is known as multi-carrier CDMA.

In a MC-CDMA system the post-detection SINR, considering the adoption of linear receivers, may be generically expressed for the \( i \)th user, \( k \)th sub-carrier as [7]:

\[
\delta_i(k) = \frac{p_i(k) h_i(k) g(k) |\mathbf{d}_i(k)|^2 H \mathbf{A} s_i(k)}{\mathcal{L}(k) + \mathcal{N}_r(k) + \sigma^2 g(k)||\mathbf{A}^T \mathbf{d}_i(k)||^2}
\]

where \( g(k) = |g(k)|^2 \) is the channel power gain between the single FRS and BS; note that the MAI is amplified at the FRS and forwarded to the BS:

\[
\mathcal{I}_r(k) = g(k) \left( \sum_{j=1}^{U} p_j(k) h_j(k) |\mathbf{d}_j(k)| H \mathbf{A} s_j(k) \right)^2
\]

and \( \mathcal{N}_r(k) \) is the normalized noise power at the BS treated through the linear multi-user receiver (MuR):

\[
\mathcal{N}_r(k) = \left( \sum_{i=1}^{U} p_i(k) h_i(k) + \mathcal{F}_r(k) \sigma^2 \right) \mathbb{E}[|\mathbf{d}_i(k)||^2]
\]

where \( p_r \) is the allocated power, and \( \sigma^2 \) is the power noise associated to the respective users (i) and sub-carrier (k).

### A. Receiver Design

Since the multiple access interference is a limiting factor in CDMA systems Verdu developed the idea of multi-user detection that takes into account the MAI information in the detection process [8]. The optimal multi-user detector is a powerful tool for MAI mitigation despite its exponential complexity, hence deploying such technique in telecommunications systems is not viable. In order to avoid complexity issues, sub-optimal linear multi-user filters [9] such as the Decorrelor (DE), Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) may be used. As shown in [6], [10] the DE is the only one that allows the power control algorithm to run distributed without demanding more overhead information to be exchanged in the system pilot channels wherefore it was chosen as the receiver design in this paper.

According to [4] the DE depends only on the user spreading codes \( (\mathbf{s}_k) \) and the correlation matrix \( (\mathbf{R}) \), and both parameters are constant during the optimization window. The DE filter is given by:

\[
\mathbf{d} = \left[ d_1, d_2, \ldots, d_U \right] = \mathbf{S} (\mathbf{S}^T \mathbf{S})^{-1} \mathbf{SR}^{-1}
\]

where \( \mathbf{S} = [s_1, s_2, \ldots, s_U] \).

### III. ENERGY AND SPECTRAL EFFICIENCIES AND PROBLEM FORMULATION

The spectral efficiency (SE) of each user through the \( N \) sub-channels can be computed as the number of bits per second that may be transmitted for a single Hertz of bandwidth. Thus, considering a practical approach for the theoretical bound obtained through the Shannon channel capacity equation, the SE of the \( i \)th user can be defined as:

\[
S_i = \frac{1}{N} \sum_{k=1}^{N} \log_2 \left( 1 + \delta_i(k) \right), \quad i = 1, \ldots, U \left[ \frac{\text{bits}}{8 \times \text{Hz}} \right]
\]

where \( i \) and \( k \) are the users and sub-channel indexes; \( \delta_i \) is the post-detection SINR given by (7). Finally, the user rate at each sub-channel is given by:

\[
r_i(k) = \mathbb{W} \cdot S_i = r_c \log_2 \left( 1 + \delta_i(k) \right) \left[ \frac{\text{bits}}{\text{s}} \right]
\]

In MC-CDMA systems with a single FRS, the energy efficiency (EE) function for user \( i \) can be formulated as [11]:

\[
\xi_i = \sum_{k=1}^{U} r_i(k) \delta_i(k) \cdot \frac{f(\delta_i(k))}{g p_i(k) + \gamma_0 p_k + \gamma_r p_r + \gamma_s}
\]

\[
\forall i, j, k \in \{1, \ldots, U\}, \text{where } i \text{ and } k \text{ are the user and sub-channel indexes respectively}; \ell = L_i(k) / V_i(k) \leq 1 \text{ is the codification rate, i.e., the number of information bits transmitted } L_i(k) \text{ divided by the total}
number of bits transmitted in a packet $V_i(k)$. The powers $p_i(k)$ is the mobile terminal transmission power, $p_b$ is the re-transmission power from the relay to the base station, assumed a fixed and equal power consumption at MT and FRS, respectively; $\varrho > 1$ and $\varrho_b > 1$ are the power amplifier inefficiency factors at MT and FRS, respectively. The efficiency function $f(\delta_i(k))$ expresses the probability of error-free packet reception. Assuming M-QAM square constellation modulations of order $M = M_i$ and gray coding the bit error rate is [12]:

$$\text{BER}_i(k) = \frac{2(\sqrt{M} - 1)}{M \log_2(M)} \times \sqrt{\frac{\delta_i(k)}{M - 1}}$$

Note that Eq. (14) is too complex to be used in real systems. According to [13] it can be approximated by:

$$\text{BER}_i(k) = \frac{2(\sqrt{M} - 1)}{M \log_2(M)} \left(1 - \sqrt{\frac{3\delta_i(k)\log_2(M)}{2(M - 1) + 3\delta_i(k)\log_2(M)}}\right)$$

which keeps the error-free packet reception function form and its behavior when $\delta \to 0$ and $\delta \to \infty$.

A. Problem Formulation

In order to maximize the energy efficiency of each user in the MC-CDMA system with a single FRS, the following problem considering the overall EE maximization with MT’s maximal power constraint is posted:

Maximize $U_i(p_i)$ subject to:

$$\sum_{i=1}^{U} \xi_i = \sum_{i=1}^{U} \sum_{k=1}^{N} r_i(k)\xi_i f(\delta_i(k))$$

s.t. (c.1) $0 \leq p_i(k) \leq P_{max}$, $i = 1, \ldots, U$  
(c.2) $\delta_i(k) \geq \delta_{min}(k)$,  $\forall k, i$

where the total transmit power of the $U$ mobile terminals across $N$ subcarriers must be bounded (and be nonnegative) for any feasible power allocation policy, with the corresponding power allocation matrix described by:

$$P \in \Phi \egal \{\{p_i(k)\}_{i \times N} | p_i(k) \geq 0, \quad p_i(k) \leq p_{i,max}\}$$

where $p_{i,max}$ represents the maximum total transmit power per subcarrier available at each MT transmitter.

IV. GAME THEORETIC APPROACH

In order to solve (16) we consider $N$ games that are played within each sub-carrier and aim to find the best SINR response for a given interference level such that each power control game is mathematically described as:

$$G(k) = \{\mathcal{U}, \{A_i(k)\}, \{u_i^k\}, k = 1, 2, \ldots, N\}$$

where $\mathcal{U} = \{1, 2, \ldots, U\}$ is the player set, $\{A_i(k)\} = [0, p_{max}, i]$ is the strategy set for user $i$ in the $k$th sub-channel with $p_{max}$, is the maximal resource (transmission power) available at the $i$th MT; and $\{u_i^k\}$ is the utility function for the $i$th user at $k$th sub-carrier; in this case $\{u_i^k\}$ is given by:

$$u_i^k = r_i(k)\xi_i \frac{(1 - \text{BER}_i(k))^{\nu(k)}}{p_i(k) + \varrho \varrho_b p_b + \varrho c + \varrho c_b}$$

with $\text{BER}_i(k)$ defined as Eq. (15). To accomplish the resource allocation in a totally distributed fashion one must define the following vector:

$$p_{-i}(k) = [p_1(k), \cdots, p_{i-1}(k), p_{i+1}(k), \cdots, p_U(k)]$$

which is the allocated power vector considering all users but user $i$. Therefore, given the power allocated to all users but $i$ at the $k$th sub-carrier the best response for user $i$ in a non-cooperative fashion may be expressed as:

$$p_i^*(k) = \text{arg max}_{P_i(k)} u_i^k[p_i(k), p_{-i}(k)]$$

Hence, the distributed energy-efficient power allocation problem under non-cooperative game perspective may be posed as:

$$\text{arg max}_{P_i(k)} u_i^k = r_i(k)\xi_i \frac{(1 - \text{BER}_i(k))^{\nu(k)}}{p_i(k) + \varrho \varrho_b p_b + \varrho c + \varrho c_b}$$

s.t. (c.1) $0 \leq p_i(k) \leq P_{max}$  
(c.2) $\delta_i(k) \geq \delta_{min}(k)$,  $\forall k, i$

Since function $u_i^k$ in Eq. (19) depends on both the user allocated power and its SINR from Eq. (7) follows the relation:

$$p_i(k) = \delta_i(k) \frac{I_i(k) + N_i(k) + \sigma^2 g(k)}{F_i(k) - \xi_i(k) g(k)} \Rightarrow \delta_i(k) = \delta_i(k)$$

where $\Gamma_i(k)$ is the sum of the MAI, the noise from the FRS and the noise at the BS, all of them normalized by the processing gain $F_i(k)$ and channel power gains of MT-FRS and FRS-BS links. On non-cooperative scheme both the MAI and the noise forwarded to the BS are considered constant during the optimization window. The fact that the power domain is an interval, i.e. $p_i(k) \in [p_{min}, p_{max}]$, and the relation between power and SINR is linear over the optimization window as shown in Eq. (23) the SINR domain is also an interval such that $\delta_i(k) \in [\delta_{min}, \delta_{max}]$, where $\delta_{min}$ is related to the SINR value when transmitting with the lowest power level and $\delta_{max}$ when transmitting with the highest power level allowed. Therefore, we can rewrite the utility function as:

$$u_i^k = r_i(k)\xi_i \frac{(1 - \text{BER}_i(k))^{\nu(k)}}{\varrho \delta_i(k) \Gamma_i(k) + \varrho_c p_b + \varrho c + \varrho c_b}$$

Note that finding the best response strategy for each user is the same as maximize the utility function (21). Its well known that functions maxima have a null derivative such that applying the derivative in equation (24), regarding $\delta_i(k)$, one may obtain:

$$\frac{\partial u_i^k}{\partial \delta_i(k)} = 0$$

Note that for concave or quasiconcave functions the best response for each user corresponds to the point where this condition is satisfied. In order to verify this we introduce the quasiconvexity concept defined as [14]-[16]:

Definition 1: (Quasiconvexity). A function $z$, that maps a convex set of $n$-dimensional vectors $\mathcal{D}$ into a real number is quasiconcave if for any $x_1$ and $x_2 \in \mathcal{D}$, $x_1 \neq x_2$, $z(x_1) \geq t$ and $z(x_2) \geq t$ for any real $t$ the following inequality is satisfied:

$$z(\lambda x_1 + (1 - \lambda)x_2) \leq t$$

In order to prove the quasi-concavity of (24) one must prove: first - that Eq. (24) numerator is S-shaped; second - that the ratio between a S-shaped function and an affine positive function is quasi-concave. For sake of simplicity, from now on, the user and sub-channel indexes are omitted from the equations.

Lemma 1: (S-shaped Numerator). The numerator in (24) is a S-shaped function.
Proof: Without loss of generality, note that Eq. (24) numerator is a function \( g(\delta) \) of the form:

\[
g(\delta) = g_1(\delta) (g_2(\delta))^V
\]  

(27)

Accordingly to [16], in order to be S-shaped a function must hold six characteristics:

1. Its domain is the interval \([0, \infty)\);
2. Its range is the interval \([0, B]\), with \(B > 0\);
3. Its Increasing;
4. It is strictly convex in the interval \([0, \delta_m]\), with \(\delta_m\) a positive number;
5. It is strictly concave in the interval \([\delta_m, L]\), with \(L\) a positive number greater than \(\delta_m\);
6. It has a continuous derivative.

Observe that 1) follows directly from \(\delta \geq 0\) and, since both \(g_1(\delta)\) and \(g_2(\delta)\) are increasing functions, then 2) and 3) are also true, even if \(B \to \infty\). Characteristics 4) and 5) are the hardest ones to be shown. To show both one must find \(\delta_m\), i.e. the function inflection point, which is given by: \(\delta_m \iff g''(\delta) = 0\). Hence, function \(g\) second derivative is given in Eq. (28) at the top of the next page. A closed expression to \(\delta_m\) was found using the least squares method and given by:

\[
\delta_m = \frac{V}{5M}
\]  

(29)

Therefore, its possible to observe properties 4 and 5, since the function is convex in the interval \([0, \frac{V}{5M}]\) and concave on the interval \([\frac{V}{5M}, B]\). One can verify 6 by simply analyzing the critical point \(\delta = 0\):

\[
\lim_{\delta \to 0^+} g(\delta) = \lim_{\delta \to 0^+} g_1(\delta) \times \lim_{\delta \to 0^+} g_2(\delta) = 0 \times \left(\frac{(V-1)}{\sqrt{M \log_2(M)}}\right)^V = 0
\]  

(30)

hence \(g(\delta)\) is continuous on the interval \([0, B]\) with \(B > 0\). This concludes the proof that \(g(\delta)\) is a S-shaped function.

Now one can prove utility function quasi-concavity based on the proof in [16]:

**Theorem 1**: (Quasiconcavity of \(u\)). The utility function \(u\) is quasi-concave in \(\delta\).

Proof: For convenience let \(u(\delta) = g(\delta)/(b(\delta))\) where \(b(\delta)\) is an affine positive function and \(u(\delta') = P^*\). Let \(t \in (0, P^*)\). Note that verifying the quasi-concavity for \(t\) outside the interval is trivial. Now, suppose that \(0 \leq \delta_1 \leq \delta_2, u(\delta_1) \geq t\) and \(u(\delta_2) \geq t\). Since \(u(\delta)\) is continuous and strictly increasing in the interval \([0, \alpha]\), there is an \(\delta_i\) such that \(u(\delta) \geq t\) for all \(\delta_i \leq \delta \leq \delta_i'\) and \(u(\delta) < t\) for \(\delta < \delta_i\). On the other hand, once \(u(\delta)\) is continuous and strictly decreasing over \((\delta_i', \alpha]\), there is an \(\delta_i''\) such that \(u(\delta) \geq t\) for \(\delta < \delta_i' \leq \delta_i''\) and \(u(\delta) < t\) for \(\delta > \delta_i''\). Then, any \(\delta\) for which \(u(\delta) \geq t\) is true, \(\delta\) must between \(\delta_i''\) and \(\delta_i''\). Similarly for any \(\delta\) such that \(\delta_i'' \leq \delta \leq \delta_i''\) then \(u(\delta) \geq t\), i.e. \(\delta \in [\delta_i', \delta_i''] \implies u(\delta) \geq t\). Therefore, \(u(\delta_1) \geq t\) and \(u(\delta_2) \geq t\) implies that \(\delta_2 \leq \delta \leq \delta_i''\) and for \(\alpha \in (0, 1)\), \(\delta_1 < \alpha \delta_1 + (1-\alpha) \delta_2 \leq \delta_i''\). Thus, \(\delta_i' < \alpha \delta_1 + (1-\alpha) \delta_2 < \delta_i''\), which implies \(u(\alpha \delta_1 + (1-\alpha) \delta_2) \geq t\).

\(^2\)The true expanded form of the second derivative is not shown here due to lack of space.

\[
\frac{\partial^2 u(\delta)}{\partial \delta^2} = \frac{\partial^2}{\partial \delta^2} \left(\frac{g(\delta)}{b(\delta)}\right) = \frac{g'(\delta) (b(\delta) - g(\delta) b'(\delta))}{(b(\delta))^2} = \frac{g'(\delta) (\delta^2 \Gamma + \varrho \Gamma + \varrho R + \varrho p_c + p_a n) - g(\delta)(\delta^2 \Gamma)}{(\rho \delta \Gamma + \varrho \Gamma + \varrho R + \varrho p_c + p_a n)^2}
\]  

(31)

To find the best SINR response and, simultaneously, allocate the correspondent transmission power level Algorithm 1 is proposed: With this framework we propose two different methods to solve step

**Algorithm 1 Iterative EE-Maximization Algorithm**

**Input**: \(p, I, \epsilon\) ; **Output**: \(p^*\)

begin
1. initialize first population and set \(n = 0\);
2. while \(n \leq I\) or \(\epsilon > \delta\) do
3. find \(\delta_i(k), \forall i = 1, \ldots, U\) \(k = 1, \ldots, N\) through (25)
4. allocate \(p_i(k)\) for all \(i\) and \(k\) in order to achieve \(\delta_i(k)\)
5. calculate \(\epsilon = \mathbb{E} (|p_i(n) - p_i(n - 1)|^2)\)
6. \(n = n + 1\)
7. end while

end

\(\mathbf{p} = \text{initial power vectors};\)
\(\mathbf{p}[n] = \text{power vector at the} n\text{th iteration};\)
\(\mathbf{p}^* = \text{power vector solution};\)
\(I = \text{maximum number of iterations};\)
\(\epsilon = \text{least expected precision};\)
\(\mathbb{E}[.] = \text{the mathematical expectation.}\)

4 in Algorithm 1: the first one uses the iterative waterfilling algorithm [17], [18] to allocate the power in order to achieve the best SINR response and the second one implements the distributed power control algorithm based on Verhulst [1], [2] equilibrium on each sub-channel. Each algorithm will be presented on the following subsections.

**A. Water-filling Algorithm**

In order to find the best power allocation for a given rate profile, i.e. the user total transmission rate \(r_i = \sum_{k=1}^{N} r_i(k)\), the water-filling algorithm was used in a Gauss-Siedel fashion as posed below [18]: The water-filling operator in Algorithm 2 is applied to each sub-

**Algorithm 2 IWFA**

**Input**: \(p, N\) ; **Output**: \(p^*\)

begin
1. initialize first population and set \(n = 0\);
2. while \(n \leq I\) do
3. for \(i = 0\) until \(U\) do
4. if \(n \mod U = 0\) then
5. \(\mathbf{p}[n + 1] = \text{WF}(\mathbf{p}[n], \mathbf{p}_i[n])\)
6. else
7. \(\mathbf{p}[n + 1] = \mathbf{p}[n]\)
8. end if
9. end for
10. \(n = n + 1\)
11. end while

end

\(\mathbf{p} = \text{initial power vectors};\)
\(\mathbf{p}^* = \text{power vector solution};\)
\(I = \text{maximum number of iterations};\)

carrier of each user considering the interference of all \(U - 1\) users, and is defined as [3]:

\[
\text{WF}(\mathbf{p}[n], \mathbf{p}_i[n]) = (\mu_a a_i - b_i)^+ \quad \forall i = 1, \ldots, U
\]  

(33)

with \((\cdot)^+ = \max(0, \cdot)\), \(\mu_i\) is the water-level that satisfy the rate constraints, and \(a_i, b_i\) are arbitrary positive numbers. Given a set of pairs \(\{(a_i, b_i)\}\) for each user in the system and a constraint function
\[ g''(\delta) = g''(\delta)(g_2(\delta))^{V} + 2V g'_1(\delta)g_2(\delta)(g_2(\delta))^{(V-1)} + V g_1(\delta)g''_2(\delta)(g_2(\delta))^{(V-1)} + V(V-1)g_1(\delta)(g'_2(\delta))^2(g_2(\delta))^{(V-2)} \] (28)

\( g \) the water-level may be obtained through the practical Algorithm 3, given in [3] with the following particularizations:

\[ a_i = 1; \quad b_i = \Gamma_i(k); \quad \zeta(\mu_i) = \prod_{k=1}^{N} \mu_i(\Gamma_i(k))^{-1} - 2^{\left(\frac{n}{2}\right)} \]

\[ \mu_i = \left[ 2^{\frac{\delta}{\zeta}} \prod_{k=1}^{N} (\Gamma_i(k)) \right]^{\frac{1}{\Delta}} \]

with \( \Gamma_i(k) \) defined in (23).

Algorithm 3 Practical algorithm for single water-filling solution

Input: set of pairs \( \{a_i, b_i\} \), function \( \zeta \); Output: \( p_i^* \)

1. set \( \tilde{N} = N; \)
2. sort \( \{a_i, b_i\} \) such that \( a_i/b_i \) are in decreasing order;
3. define \( \eta_{N+1} = 0; \)
4. while \( b_{N_1}/a_{N_1} \geq b_{N+1}/a_{N+1} \) or \( \zeta(b_{N}/a_{N}) \geq 0 \)
   set \( N = N - 1; \)
   end while
5. find \( \mu_i \in (b_{N}/a_{N}, b_{N+1}/a_{N+1}) \) if \( \zeta(\mu_i) = 0 \)
6. \( x_i = (\mu_i/\alpha_i - b_i)^+; 1 \leq i \leq \tilde{N} \)

\( (x)^+ = \max(x, 0); \)
\( \tilde{N} \) is number of active sub-carriers;

B. Verhulst DPCA

The Verhulst mathematical model was first idealized to describe population dynamics based on food and space limitation. In [1] that model was adapted to single-rate DS/CDMA distributed power control and further extended to multi-rate systems in [2] using a discrete iterative convergent equation as follows:

\[ p_i(k)[n+1] = (1 + \alpha) p_i(k)[n] - \alpha \left( \frac{\delta_i(k)[n]}{\delta_i^*(k)} \right) p_i(k)[n], \quad i = 1, \ldots, U \]

(38)

where \( p_i(k)[n+1] \) is the \( i \)th user \( k \)th sub-carrier power updated at the \( n+1 \) iteration and is bounded by \( 0 \leq p_i[k][n+1] \leq p_i[\text{max}]; \alpha \in (0; 1) \) is the Verhulst convergence factor; \( \delta_i(k)[n] \) is the \( i \)th user \( k \)th sub-carrier SINR at iteration \( n \), and \( \delta_i^*(k) \) is the minimum SINR that satisfy the best SINR response in Algorithm 1. Since it was first designed for DS/CDMA networks, the DPCA Verhulst will allocate the transmission power individually on each sub-channel.

V. NUMERICAL EXAMPLE

In order to verify which of the proposed methods has the best performance in maximizing the energy efficiency simulations were conducted using MatLab Platform 7.0. The simulations parameters are shown in Table I.

In order to illustrate the quasi-concavity of the utility function Figure 2 is presented. It shows the value of \( u_1^* \) for different values of \( \delta_1(1) \). Note that as expected the function has only one maximizer point (red circle), which is achieved by the IWFA in this case. To

**Table I**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Adopted Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MC-CDMA System</strong></td>
<td></td>
</tr>
<tr>
<td>Noise Power ( P_n )</td>
<td>-90 [dBm]</td>
</tr>
<tr>
<td>Circuitry Power ( P_c )</td>
<td>1 [W] per user</td>
</tr>
<tr>
<td>Relay Circuitry Power ( P_R )</td>
<td>5 [W]</td>
</tr>
<tr>
<td>Relay Transmission Power ( P_R )</td>
<td>30 [W]</td>
</tr>
<tr>
<td>Power Amplifier Inefficiency ( \eta )</td>
<td>2.5</td>
</tr>
<tr>
<td>Codification Rate ( \epsilon )</td>
<td>3</td>
</tr>
<tr>
<td>Sub-carriers ( N )</td>
<td>16</td>
</tr>
<tr>
<td>Amplification Matrix ( A )</td>
<td>( \frac{F}{10^9} ) [5]</td>
</tr>
<tr>
<td>Sub-channel Bandwidth ( w )</td>
<td>( 1 \times 10^9 ) Hz</td>
</tr>
<tr>
<td>Max. power per user ( P_{\text{max}} )</td>
<td>2 [W] per sub-channel</td>
</tr>
<tr>
<td># mobile terminals ( I )</td>
<td>5</td>
</tr>
<tr>
<td># base station ( BS )</td>
<td>1</td>
</tr>
<tr>
<td># fixed relay station ( FRS )</td>
<td>1</td>
</tr>
<tr>
<td>cell geometry</td>
<td>rectangular</td>
</tr>
<tr>
<td>mobile term. distrib.</td>
<td>( x_{\text{cell}} = 10 \text{Km} )</td>
</tr>
<tr>
<td></td>
<td>( y_{\text{cell}} = 5 \text{Km} )</td>
</tr>
</tbody>
</table>

**Channel Gain**

path loss \( \alpha \cdot d^{-2} \)
shadowing uncorrelated log-normal, \( \sigma^2 = 6 \text{ dB} \)
fading Rayleigh

<table>
<thead>
<tr>
<th>User Types</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>User Rates</td>
<td>([0.8; 1.5; 3] \times w ) [bps]</td>
</tr>
<tr>
<td></td>
<td>( M = [4\text{-QAM}; 16\text{-QAM}; 64\text{-QAM}] )</td>
</tr>
</tbody>
</table>

**Verhulst Power-Rate algorithm**

<table>
<thead>
<tr>
<th>Type</th>
<th>distributed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.5 *</td>
</tr>
<tr>
<td>Optimization window</td>
<td>50 passes</td>
</tr>
</tbody>
</table>

**IWFA**

<table>
<thead>
<tr>
<th>Type</th>
<th>Gauss-Seidel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. # iterations</td>
<td>100 iterations</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>( 10^{-6} )</td>
</tr>
</tbody>
</table>

**EE Maximization Algorithm**

<table>
<thead>
<tr>
<th>( I )</th>
<th>100 iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon )</td>
<td>( 10^{-6} )</td>
</tr>
</tbody>
</table>

Fig. 2. Utility Function for user #1 at sub-carrier #1. Parameters: 4-QAM, \( V = 1 \), \( r_{1, \text{min}} = 0.8 \text{Mbps} \).
show both approaches performance we introduce first results for EE and later for SE, under the same conditions. Note in Figure 3 that the IWF A achieves higher values of EE for most users, and in fact the total EE for the water-filling approach is 10% greater than the Verhulst one. This can be easily justified due to Verhulst DPCA which does not permit the optimization process to be hold through all sub-carriers at the same time, while the water-filling algorithm holds this characteristic. In terms of spectral efficiency the IWF A also presents a slight gain when compared to the Verhulst DPCA. Figure 4 shows the sum rates for both algorithms compared to the minimum rate of each user. Note that IFWA achieves a SE of 1.17bits/s/Hz while the Verhulst DPCA 0.98bit/s/Hz.

VI. CONCLUSIONS

The EE maximization in MC-CDMA cooperative wireless networks is a NP-hard optimization problem of great interest for future networks systems as well as wireless sensor networks. In this paper we presented a game theoretic approach of this problem and two different power control algorithms that can be easily deployed on the inner loop of Algorithm 1. The first one is the simplest solution for power control in MC-CDMA networks and the second one is a distributed power control algorithm firstly design for DS/CDMA networks and herein adapted to the MC-CDMA power control. The superiority of the IFWA over DPCA Verhulst is evident in the numerical results both in terms of EE and SE. This may be explained through the fact that the IFWA optimizes the power allocation of each user through all sub-carriers at the same time while on the other hand Verhulst approach performs the power control on each sub-channel considering all users. Future work includes a complexity analysis in order to determine if the circuitry power of both IFWA and Verhulst DPCA may be considered equivalent.

REFERENCES