Improvement of MISO Single-user Time Reversal Ultra-wideband Using a DFE Channel Equalizer

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Abstract—In this paper a baseband multiple-input single-output (MISO) time reversal ultra-wideband system (TR-UWB) incorporating a decision feedback equalizer (DFE) is evaluated over the scenarios CM1 and CM3 of the IEEE 802.15.3a channel model. A semi-analytical performance based on a Gaussian approximation is derived and compared with simulation results. The results show that such approach represents a good approximation for the bit error rate (BER) analysis, and that the DFE, as well as an increase in the number of transmit antennas, improve the system performance.

I. INTRODUCTION

Ultra-wideband has been considered as a promising solution for short distance high data rate communications, such as wireless personal area networks and sensor networks applications. Due to the very large bandwidths typically used, the UWB channel is characterized by a dense multipath environment. In order to effectively capture the energy spread over the multipath components, the transmit based time reversal (TR) technique has been investigated [1], [2]. In baseband time reversal, the channel impulse response (CIR) is estimated from a probe signal, and the data is convolved with the complex conjugate time reversed version of the estimated CIR (time reversal coefficients) prior to transmission. TR-based UWB transmission can provide inter-symbol interference (ISI) mitigation by reducing the delay spread of the channel, and also co-channel interference rejection by focusing the signal on the point of interest. TR is based on the channel reciprocity, which was experimentally verified in [1] for a particular UWB environment. In [1] the authors also mention that a space of 20 cm is enough to guarantee no correlation between antennas in a particular MISO UWB environment.

However, for high transmission rates, the residual ISI will still degrade the system performance, because the equivalent channel impulse response after TR is not a delta function. In addition, if the channel multipaths are correlated or there is a slight change in the environment over the propagation path, additional ISI will be introduced. To handle such impairments, a receive based channel equalization scheme can be employed with fewer taps than that used without TR [3], [4].

This work considers an equivalent baseband time reversal system with multiple transmit antennas and channel equalization at the receiver side. The transmission is from base station with relatively good computational capacity to lower complex device with hardware constraints. Independent channel realization across antennas, perfect channel knowledge at the transmitter, and perfect synchronization at the receiver are ideally assumed.

II. SYSTEM MODEL

The pulse filter has a square-root raised cosine shape [5]

\[
\begin{align*}
g_T(t) &= \begin{cases} 
\frac{1}{\sqrt{T}} \sin[(1-\alpha)t/T] + (4\alpha t/T) \cos[(1+\alpha)t/T], & t \neq 0, t \neq \frac{T}{4\alpha} \\
\frac{1}{\sqrt{T}} \left[1 - \alpha + \frac{2\alpha}{\pi} \right], & t = 0 \\
\frac{1}{\sqrt{2T}} \left[(1+\alpha) \sin(\frac{\alpha}{T}) + (1-\frac{\pi}{2\alpha}) \cos(\frac{\alpha}{T}) \right], & t = \pm \frac{T}{4\alpha}
\end{cases}
\end{align*}
\]

where \( T \) is the reciprocal of the symbol rate, and \( \alpha \) is the roll-off factor. In this work the parameter \( T \) is kept constant for the pulse shape generation, but the effective symbol rate is controlled by the space between consecutive symbols given by \( T_s = \kappa T \), where \( \kappa \) is an integer. For an antipodal binary signaling with symbols \( b_i \in \{\pm 1\} \), the signal before time reversal convolution is given by

\[
s(t) = \sum_{i=-\infty}^{\infty} b_i g_T(t - iT_s)
\]

The single-user MISO TR-UWB system is presented in Figure 1. Assuming perfect channel estimation, \( \hat{h}_k(t) \), on the \( k \)th antenna, the signal to be transmitted, \( s(t) \), is convolved with the TR signal \( h_k^R(t) = h_k^R(-t) \). If the channel is perfectly reciprocal and slowly varying, the equivalent baseband received signal can be represented as

\[
r(t) = s(t) \ast \sum_{k=1}^{A_k} h_k^R(t) \ast h_k(t) + \eta(t)
\]
where $A_t$ is the number of antenna elements, $\eta(t)$ represents the equivalent complex additive white Gaussian noise (AWGN), and $*$ denotes the convolution operator.

### III. SIGNAL-TO-INTERFERENCE-PLUS-NOISE RATIO ANALYSIS

The received signal is subjected to a filter $g_R(t)$ matched to the pulse $g_T$. The output of the matched filter is

$$y(t) = s(t) * \sum_{k=1}^{A_t} h_{TR}^*(t-k) * h_k(t) * g_R(t) + \eta(t) * g_R(t)$$

(4)

Substituting Equation (2) in (4)

$$y(t) = \sum_{i=-\infty}^{\infty} b_i g_T(t-iT_s) * \sum_{k=1}^{A_t} h_{TR}^*(t-iT_s-k) * h_k(t) * g_R(t) + z(t) + \eta(t) * g_R(t)$$

(5)

A. Pure Time Reversal

When the output of the matched filter is sampled at a rate $1/T_s$, it becomes

$$y(nT_s) = \sum_{i=-\infty}^{\infty} b_i x([n-i]T_s) + z(nT_s)$$

(6)

With the change of variable, $v = n - i \Rightarrow i = n - v$, it follows that

$$y_n = \sum_{v=-\infty}^{\infty} b_{n-v} x_v + z_n$$

(7)

where $y_n = y(nT_s)$, $z_n = z(nT_s)$, and $x_v = (vT_s)$. The notation $x_0$ represents the peak of the autocorrelation function $x_v$, which is assumed to be perfectly synchronized at the receiver. For this complex baseband representation with complex transmission and perfect CIR knowledge at the transmitter, $x_0$ is real at the receiver, while the residual ISI is still complex. Note that the discrete-time sequence that represents the sampled noise, $z_n$, is still AWGN, and its variance (or power) is defined as $\sigma_z^2$. Hence, the variance of the in-phase and quadrature components of $z_n$ are equal and given by $\sigma_z^2 = \sigma_d^2/2$ [6]. The decision variable is $V = \Re\{y_n\}$. The signal-to-interference-plus-noise ratio (SINR) conditioned on the $j$th set of channel realizations can be obtained as

$$\text{SINR}_{tr}^j = \frac{\mathbb{E}\{\Re\{b_n x_n\}\}^2}{\mathbb{E}\{\sum_{v=-\infty}^{\infty} \Re\{b_{n-v} x_v\}\}^2 + \sigma_z^2},$$

(8)

where $\mathbb{E}\{\cdot\}$ denotes the expected value operator. If the information symbols are independent and identically distributed (i.i.d.) and $\sigma_d^2 = \mathbb{E}\{b^2\} = 1$, Equation (8) can be rewritten as

$$\text{SINR}_{tr}^j = \frac{\Re\{x_n^j\}^2}{\Re\{x_v^j\}^2 + \sigma_z^2},$$

(9)

B. Time Reversal With DFE Channel Equalization

The DFE is a non-linear equalizer that employs a feedforward filter $C(f)$ and a feedback filter $D(f)$. Generally, the feedforward filter is a linear minimum square error (MSE) equalizer that partially eliminates the ISI. The feedback filter is used to remove that part of the intersymbol interference from the present estimate caused by previously detected symbols. Since the feedback filter assumes an error free signal as its input, such filter mitigates the ISI without introducing noise into the system. However, when a wrong decision is fed back, error propagation is introduced, which reduces the system performance. Figure 2 illustrates the basic DFE structure.

![DFE structure](image-url)

Due to the time reversal procedure, the imaginary part of the interest signal in (7) is negligible. Therefore, the input of the equalizer can be set as $y_n' = \Re\{y_n\}$. Based on [7], the optimum taps of the feedforward section $C(f)$ can be determined as
\[
\sum_{m=0}^{N_f-1} \sum_{u=0}^{l} c_m x_{u+m-l} \Delta b, \quad l = 0 \cdots N_f - 1, \quad (10)
\]
and
\[
d_k = \sum_{m=0}^{N_f-1} c_m x_{m+k}, \quad k = 1 \cdots N_b \quad (11)
\]
for the feedback filter, where \( x'_u = \Re \{ x_u \} \). The approach to calculate the \( \text{SINR} \) at output of the DFE is based on [8], but here the equalizer length is not longer than the equivalent CIR. The equalized signal \( b_n \) is given by
\[
b_n = \sum_{m=0}^{N_f-1} c_m y_{n+m} - \sum_{k=1}^{N_b} d_k \tilde{b}_{n-k} \quad (12)
\]
Substituting \( y'_n \) into Equation (12), it becomes
\[
\tilde{b}_n = \sum_{m=0}^{N_f-1} c_m \left( \sum_{v=-\infty}^{\infty} b_{n+m-v} x'_v + z'_n \right) - \sum_{k=1}^{N_b} d_k \tilde{b}_{n-k}, \quad (13)
\]
where \( z'_n = \Re \{ z_n \} \). After the change of variables \( p = v - m, \quad v = p + m, \) and noting that \( v \rightarrow \pm \infty, \quad p \rightarrow \pm \infty, \)
\[
\tilde{b}_n = \sum_{m=0}^{N_f-1} c_m \left( \sum_{p=-\infty}^{\infty} b_{n+p-m} x'_p + z'_n \right) - \sum_{k=1}^{N_b} d_k \tilde{b}_{n-k}
= \sum_{p=-\infty}^{\infty} \sum_{m=0}^{N_f-1} c_m x'_{p+m-n} b_{n+p-m} - \sum_{k=1}^{N_b} \sum_{m=0}^{N_f-1} c_m z'_{n+m} - \sum_{k=1}^{N_b} d_k \tilde{b}_{n-k} \quad (14)
\]
Defining
\[
f_p = \sum_{m=0}^{N_f-1} c_m x'_{p+m},
\]
Equation (14) results in,
\[
\tilde{b}_n = \sum_{p=-\infty}^{\infty} f_p b_{n-p} + \sum_{m=0}^{N_f-1} c_m z'_{n+m} - \sum_{k=1}^{N_b} d_k \tilde{b}_{n-k} \quad (16)
\]
The first summation in (16) can be divided into four parts
\[
\tilde{b}_n = f_0 b_n + \sum_{p=-\infty}^{-1} f_p b_{n-p} + \sum_{p=0}^{N_b} f_p b_{n-p} + \sum_{p=1}^{\infty} f_p b_{n-p} + \sum_{p=0}^{N_f-1} c_m z'_{n+m} - \sum_{k=1}^{N_b} d_k \tilde{b}_{n-k}
\]
Assuming there are no feedback errors, \( \tilde{b}_{n-k} = b_{n-p} \) for \( k = p \), the third and the last terms in Equation (17) are canceled, since \( d_k = f_p \). Hence,
\[
\tilde{b}_n = f_0 b_n + \sum_{p=-\infty}^{-1} f_p b_{n-p} + \sum_{p=N_b+1}^{\infty} f_p b_{n-p} + \sum_{m=0}^{N_f-1} c_m z'_{n+m} \quad (18)
\]
The first term in Equation (18) represents the desired signal, the second and the third are the residual ISI, while the last one is the resultant noise after equalization. The decision variable is given by \( V = \Re \{ \tilde{b}_n \} = \tilde{b}_n \). Hence, the \( \text{SINR} \) conditioned to the \( j \)th set of channel realization is given by Equation (19) (at the top of next page). If the information symbols \( b_n \in \{ \pm 1 \} \) are i.i.d., Equation (19) is rewritten as
\[
\text{SINR}_{df} = \frac{\{ f_{ij} \}^2}{\sum_{p=-\infty}^{\infty} \{ f_p \}^2 + \sum_{p=N_b+1}^{\infty} \{ f_p \}^2 + \sigma^2} \quad (20)
\]
C. Noise Calibration, MF Bound and BER Performance
Defining the signal-to-noise ratio (\( \text{SNR} \)) at the output of the matched filter as
\[
\text{SNR} = \frac{E_{bd}}{\sigma_d^2} = \frac{E_{bd}}{2 \sigma^2}, \quad (21)
\]
where \( E_{bd} = |x_0|^2 \) is here defined as the bit energy (peak energy) after MF, the variance of the in-phase and quadrature components of \( z_n \) are obtained as
\[
\sigma^2 = \frac{E_{bd}}{2 \text{SNR}} \quad (22)
\]
The single-user matched filter bound for an antipodal binary signaling is defined as
\[
S_{UB} = Q \left( \sqrt{2 \text{SNR}} \right) = Q \left( \sqrt{\frac{2E_{bd}}{\sigma_d^2}} \right), \quad (23)
\]
where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-y^2/2} dy \). If the denominators of the Equations (9) and (20) are assumed to be Gaussian distributed, the bit error ratio (BER) conditioned to the \( j \)th set of channel realizations can be approximated as
\[
\text{BER}^j = Q \left( \sqrt{\text{SINR}} \right) \quad (24)
\]
Considering \( J \) sets of channel realizations, the average BER can be computed as
\[
\text{BER} = \frac{1}{J} \sum_{j=1}^{J} \text{BER}^j \quad (25)
\]
The definition of \( \text{SNR} \) discussed above is based on [2] and does not emphasize the array gain of the MISO system, because the noise power is calculated according to the peak energy of the equivalent CIR at the receiver, regardless the number of transmit antennas.

IV. CHANNEL MODEL AND TIME REVERSAL FILTER
A. Channel Model
The channel modeling subcommittee of the TG3a recommended a channel model that is basically a modified version of the Saleh-Valenzuela (mSV) model [9], [10], [11], where multipath components (MPCs) arrive at the receiver in clusters. Cluster arrivals are Poisson distributed with rate \( \Lambda \). The ray arrivals within each cluster are also a Poisson process with rate \( \lambda > \Lambda \). The arrival time of the \( m \)th cluster is denoted by \( \tau_m \), and the arrival time of the \( n \)th ray within the \( m \)th cluster by \( \tau_{m,n} \). The channel coefficient gain \( \beta_{m,n} \) is described by a log-normal distribution and its phase has only values 0
or $\pi$ with equal probability. Four scenarios were proposed: CM1 – based on line of sight (LOS) 0-4m, CM2 – based on non-LOS (NLOS) 0-4m, CM3 – based on NLOS 4-10m, and CM4 based on an extreme NLOS environment. The present work considers the CM1 and CM3 scenarios. One multipath channel realization consists of cluster arriving rays:

$$h_c(t) = \chi \sum_{m=0}^{M} \sum_{n=0}^{N} \beta_{m,n} \delta(t - \tau_{m,n}),$$

where $\delta$ is the Dirac delta function and $\chi$ represents the log-normal shadowing term. The above described CIR is not a baseband complex tap model and its output is a continuous time arrival and amplitude value.

### B. Discrete-time Baseband Channel and TR Coefficients

The channel characteristics provided in [9], [10] are based on a $t_{s} = 167$ ps sampling time. An arbitrary realization of the channel with multipath resolution $t_{s}$ can be represented as the following periodic impulse train

$$h'(t) = \chi \sum_{\ell=0}^{L} \beta_{\ell} \delta(t - \ell t_{s}),$$

where $\tau_{\ell} = \ell t_{s}$. In this paper, a baseband signal analysis is adopted. Thus, since the mSV is a bandpass channel model, its complex baseband version must be generated. The discrete-time baseband CIR with sampling time $T$ is obtained by

$$h_{d}[nT] = \int_{-\infty}^{\infty} p(nT-t)h'(t)e^{-j\omega_c t}dt = \chi \sum_{\ell=0}^{L} p(nT - \tau_{\ell}) \beta_{\ell} e^{-j\omega_c \tau_{\ell}},$$

where $p(t) = g_R(t) * g_T(t)$, and $g_T(t)$ is the pulse shape defined by Equation (1) with roll-off factor $\alpha = 0.3$ and $T = 3t_{s} = 501$ ps. The down-conversion is performed by $e^{-j2\pi f_c t}$ with the carrier center frequency $f_c = 4.1$ GHz. Additionally, the channel coefficients are normalized to unity energy, which means that the shadowing factor is not taken into account. The resultant discrete-time complex baseband CIR is used in order to obtain the simulation results and also to derive the semi-analytical performances. Note that the pulse shape effect is already included in the discrete-time baseband CIR [12].

The TR filter considers that the CIR is perfectly estimated at the transmitter. Figure 3 illustrates the compression of the CIR after time reversal.

### V. Simulation Configuration and Results

Monte Carlo simulation (MCS) method is adopted with two transmission rates: 499 Mbps ($\kappa = 4$ — Rate A), and 665.34 Mbps ($\kappa = 3$ — Rate B). The signal at the output of the matched filter is sampled at a rate equal to $1/T_s$. The channel is assumed to remain static during a frame interval of duration $T_f = 100$ $\mu$s [10]. As the pulse shape filter is already included in the channel coefficients $h_d[nT]$, the signal to be transmitted is generated by simply convolving the information symbols$^2$ with the TR coefficients, and the matched filter is implemented by sampling the received sequence with a sampling time given by $T_s$. Figure 4 illustrates the equivalent simulation model. $h_{d,k}$ represents the discrete time filtered CIR for the $k$th antenna, according to Equation (28). At least 200 different sets of random choices from 100 realizations proposed in [9] are considered for the average BER calculation in each $SNR$ simulated point.

The decision feedback equalizer is implemented considering the recursive least squares (RLS) algorithm with forgetting factor set to 0.999, taps spaced by $T_s$, and a training sequence overhead of 1 $\mu$s. In the configuration Rate A, 7 and 6 taps are considered for the feedforward and feedback filter, while in the Rate B, 9 and 8 taps are considered, respectively. The semi-analytical results (THEO) are obtained considering $J = 3000$ sets of channel realizations and are compared to the MCS results. Figures 5 to 8 show the MCS and THEO BER results. The system performance gets better with an increase.

$^2$There are $(\kappa - 1)$ zero samples between each information symbol.
in the number of antenna elements and when the equalizer is considered. In both Rate A and Rate B configurations, the DFE has a better performance over the scenario CM1 than over CM3, for the same number of taps, because the equivalent CIR after TR is longer in the latter scenario. When three antennas and DFE are considered at the same time, the system has a good BER performance. Note that the semi-analytical and the MCS results are close to each other.

VI. CONCLUSIONS

This paper presented a performance analysis for a single-user MISO TR-UWB system. Due to the ideal conditions assumed here, the obtained results may be interpreted as lower bound performances. The results showed that the number of antenna elements and the presence of a DFE play a significant role on the system performance: the performance gets better when the number of antennas increases and equalization is employed. The semi-analytical results considering Gaussian assumption represent a good approximation for the BER in the analyzed cases. A relatively low error rate can be obtained when three transmit antennas are used together with a DFE for data rates up to 499 Mbps or even 665.34 Mbps.

REFERENCES


