Ultra-wideband Performance in a Dense Multipath Environment with Time and Spatial Diversity

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Abstract—This paper analyzes the performance of direct sequence ultra-wideband (DS-UWB) systems in a very dense and realistic multipath channel scenario with temporal and spatial diversity, considering Monte-Carlo simulation method and semi-analytical performance results are the same but we re
measurements include indoor and handheld applications.
vehicular radar systems. Particularly, the communications and
imaging systems, communications and measurements, and
technology. Three categories of UWB systems were specified:
(3.6-10.1 GHz). The FCC rules do not define a specific total channel energy; the shadowing effects were considered.
Each user has one specific spreading sequence and the number of Rake fingers on the system performance, emphasizing the effect of the pulse shape correlation.

I. INTRODUCTION

UWB is a emerging technology that employs baseband ultra short pulses to transmit the information, resulting in a very large bandwidth. These systems were recently been considered with great interest due to some attractive characteristics such as potential for very high data rates, low cost, low probability of interception, resistant to severe multipath and jamming, and very good time domains resolution allowing location and tracking applications at centimeter level. Some potential applications of UWB are [2]: wireless personal area networks, wireless sensor networks, imaging systems, and vehicular radar systems.

In February 2002, the Federal Communications Commission (FCC) announced the FCC First Report and Order (R&O) [3] that permitted the deployment of UWB devices for data communications over an enormous bandwidth up to 7.5GHz (3.6-10.1 GHz). The FCC rules do not define a specific technology. Three categories of UWB systems were specified: imaging systems, communications and measurements, and vehicular radar systems. Particularly, the communications and measurements include indoor and handheld applications.

This work is based on [1] with some differences, such as: the semi-analytical performance results are the same but were differently developed; the number of multipath components was not fixed and had been chosen to represent 85% of the total channel energy; the shadowing effects were considered.

The paper is organized as follows: in the Sections II and III, the signal model and the channel model are briefly described. The Section IV brings the receiver structure description and section V presents the semi-analytical BER performance analysis. To sum it up, Section VI brings the numerical results and Section VII discusses the main conclusions of the work.

II. SIGNAL MODEL

A direct sequence ultra-wideband system with $N_u$ asynchronous users and binary phase shift keying (BPSK) modulation is considered. Each user has one specific spreading sequence with $N_c$ chips, such that $T_b = T_{C}$, where $T_b$ and $T_C$ are, respectively, the bit and chip duration. The signal transmitted by the $u$th user is given by

$$s^{(u)}(t) = \sum_{i=-\infty}^{\infty} \sum_{j=1}^{N_c} b_i^{(u)} a_j^{(u)} w(t - iT_b - jT_C)$$

where $w(t)$ is the pulse shape, and $b_i$ and $a_j$ are the data symbols and the spreading chips for the $u$th user, respectively.

III. CHANNEL MODEL

The adopted channel model is the Saleh-Valenzuela (SV) [4] with slight modifications, similar to the one described in [5]. In this model, multipath components (MPC’s) arrive at the receiver in clusters. Cluster arrivals are Poison distributed with rate $\Lambda$. The ray arrivals within each cluster are also a Poison process with rate $\lambda > \Lambda$. The arrival time of the $m$th cluster is denoted by $\tau_{m}$, and the arrival time of the $n$th ray within the $m$th cluster by $\tau_{m,n}$. The $(m,n)$th gain is described by a log-normal distribution, and the $(m,n)$th phase is constrained to take only values 0 or $\pi$ with equal probability. The $u$th channel impulse response with a linear antenna array with $K$ elements can be written as

$$h^{(u)}_u(t) = \sum_{k=0}^{K-1} \sum_{m=0}^{L_{TOT}-1} \sum_{n=0}^{N-1} \alpha^{(u)}_{m,n,k} \delta(t - \tau^{(u)}_{m,n,k})$$

where $L_{TOT}$ is the total number of MPC’s (considering all $M$ clusters and $N$ rays), $\alpha^{(u)}_{m,n,k} \in \mathbb{R}$, $\chi^{(u)}$ is the shadowing factor, $\beta^{(u)}_{l,k} = \chi^{(u)} \alpha^{(u)}_{l,k}$ and $\delta(t)$ is the Dirac delta function.

We have considered a simplified (S-V) channel model with $L_{TOT}$ representing 85% of the total channel energy. The SV UWB channel model is characterized by the following parameters: a) mean and RMS spread delay, $\tau_m$ and, $\tau_{rms}$,
respectively; b) decay power profile; c) mean number of multipath components with mean squared value in the range of \([-10; 0]\) dB, related to the ray with greatest power, \(N_{P10dB}\); or alternatively, it can be considered the number of multipath components with mean power equivalent to 85% of the total power. The four channel models (CM-1 to CM-4) and its main parameters are listed in Table III [5].

### TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CM-1</th>
<th>CM-2</th>
<th>CM-3</th>
<th>CM-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_{\text{rms}}) [ns]</td>
<td>5.0</td>
<td>9.9</td>
<td>15.9</td>
<td>30.1</td>
</tr>
<tr>
<td>(\tau_{\text{rms}}) [ns]</td>
<td>5</td>
<td>8</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>(N_{P10dB})</td>
<td>12.5</td>
<td>15.3</td>
<td>24.9</td>
<td>41.2</td>
</tr>
<tr>
<td>(&lt;\beta^2&gt;) [dB]</td>
<td>20.8</td>
<td>33.9</td>
<td>64.7</td>
<td>123.3</td>
</tr>
<tr>
<td>(\sigma_{p,2}) [dB]</td>
<td>2.9</td>
<td>3.3</td>
<td>3.4</td>
<td>3.2</td>
</tr>
</tbody>
</table>

1 Sampling period of 167 ps, corresponding to the spatial resolution of 5 cm.

### IV. RECEIVER STRUCTURE

The receiver structure is the same one described in [1] (figure 1). Perfect synchronism is assumed to the desired user (with index 1) and the delays and channel coefficients of the selected paths are exactly known at the receiver. The receiver for the desired user selects the strongest \(L_f^1\) paths per antenna using a maximal ratio combining (MRC) scheme. The received signal is given by

\[
r(t) = \sum_{k=0}^{K-1} \sum_{l_1=0}^{L_f^1-1} \beta_{l_1,k} s^1(t - \tau_{l_1,k}^1) + \sum_{k=0}^{K-1} \sum_{l_1=0, l_1 \neq l_f}^{L_f^1-1} \beta_{l_1,k} s^1(t - \tau_{l_1,k}^1) + \sum_{k=0}^{K-1} \sum_{l_1=0}^{L_f^1-1} n_{\text{SI}}(t) + \sum_{k=0}^{K-1} \sum_{l_1=0}^{L_f^1-1} n_{\text{MAI}}(t) + \sum_{k=0}^{K-1} \sum_{l_1=0}^{L_f^1-1} \eta_k(t) + \sum_{k=0}^{K-1} \sum_{l_1=0}^{L_f^1-1} \eta_k(t)
\]

where \(n_{\text{SI}}(t)\) is the self-interference (SI), \(n_{\text{MAI}}(t)\) the multiple access interference (MAI), and \(\eta(t)\) the background noise. The term \(\eta_k(t)\) represents the additive white Gaussian noise (AWGN) with bilateral power spectral density given by \(N_0/2\); \(\beta_{l_1,k}^1\) and \(\tau_{l_1,k}^1\) are, respectively, the channel coefficients and the delay of the \(l_1\)th path at the \(k\)th antenna element for the user \(u\). Without loss of generality, \(\tau_{0,k,c}^1\) is assumed to be zero \((\tau_{0,k,c}^1 = 0)\), where \(Kc\) is the reference element.

### V. PERFORMANCE ANALYSIS

The energy of the transmitted symbol can be calculated as

\[
E_{\text{bit}} = \int_{-\infty}^{\infty} \left( \sum_{j=0}^{N_c-1} b_j^{(u)} a_j^{(u)} w(t - iT_b - jT_c) \right)^2 dt = N_c \int_{-\infty}^{\infty} w^2(t) dt = N_c E_w
\]

where \(E_w\) is the energy of the shaping pulse.

#### A. Desired Signal

The desired signal at the output of the \(l_f\) correlator at the \(k\)th element is given by

\[
S_{l_f,k} = \sum_{j=0}^{N_c-1} \beta_{l_f,k}^1 b_0^1 a_j^1 w(t - jT_c - \tau_{l_f,k}^1) \cdot \sum_{j=0}^{N_c-1} \beta_{l_f,k}^1 a_j^1 w(t - iT_c - \tau_{l_f,k}^1) dt + \left( \beta_{l_f,k}^1 \right)^2 b_0^1 N_s \int_0^{T_c} w^2(t) dt = \left( \beta_{l_f,k}^1 \right)^2 b_0^1 E_{\text{bit}}
\]

Therefore, the total signal energy can be written as

\[
E_S = \left( \sum_{k=0}^{K-1} \sum_{l_f=0}^{L_f^1-1} S_{l_f,k} \right)^2 = \left( \sum_{k=0}^{K-1} \sum_{l_f=0}^{L_f^1-1} \left( \beta_{l_f,k}^1 \right)^2 \right) E_{\text{bit}}
\]

#### B. Additive Noise

The correspondent AWGN term is

\[
N_{l_f,k} = \int_{-\infty}^{\infty} \eta_k(t) \sum_{j=0}^{N_c-1} \beta_{l_f,k}^1 a_j^1 w(t - jT_c - \tau_{l_f,k}^1) dt
\]

The total AWGN at the receiver output is given by

\[
\eta = \sum_{k=0}^{K-1} \sum_{l_f=0}^{L_f^1-1} N_{l_f,k} = \sum_{k=0}^{K-1} \sum_{l_f=0}^{L_f^1-1} \left[ \int_{-\infty}^{\infty} \eta_k(t) \cdot \sum_{j=0}^{N_c-1} \beta_{l_f,k}^1 a_j^1 w(t - jT_c - \tau_{l_f,k}^1) dt \right]
\]

Fig. 1. DS-UWB receiver structure.
It can be shown that $E[\eta] = 0$, and that

$$\sigma_0^2 = \frac{N_0}{2} \sum_{k=0}^{K-1} \sum_{l_j=0}^{L_l-1} (\beta_{l_j,k})^2 E_{\text{bit}}$$

(9)

C. MAI Term

The MAI contribution is represented by

$$M_{l,f,k} = \sum_{u=2}^{N_u} \sum_{l=0}^{L_u-1} (\beta_{l,k})^1 \beta_{l,f,k}.$$  

$$\int_{\tau_{l,f,k}}^{\tau_{l,f,k} + T_u} \left[ \sum_{j=0}^{N_l-1} a_j^u \left( t - jT_c - \tau_{l,j,k} \right) \right] dt$$  

(10)

In order to solve the integral $I_{l,f,k}^{u,1}$ consider figure 2. The delay between the $l$th path of the user $1$ and all paths from the $u$th user at the $k$th element is $\tau_{l,j,k} = \tau_{l,j,k}^{u,1} - \tau_{l,j,k}^{1} = \gamma_{l,j,k}^{u,1} T_c + \Delta_{l,j,k}^{u,1}$ where $\gamma_{l,j,k}^{u,1}$ represents the integer part multiple of $T_c$ and $\Delta_{l,j,k}^{u,1}$ the fractional part uniformly distributed in $[0, T_c)$.

$$I_{l,f,k}^{u,1} = \left[ \begin{array}{c}
\Delta \\
\int_{0}^{\Delta} \phi_1(a^u, a^u) \phi_2(a^u, a^u) \\
\int_{T_f}^{\Delta} \phi_1(a^u, a^u) \phi_2(a^u, a^u)
\end{array} \right]$$

(11)

Hence, the total MAI term can be written as

$$I_{\text{MAI}} = \sum_{k=0}^{K-1} \sum_{l_j=0}^{L_l-1} M_{l,f,k} = \sum_{k=0}^{K-1} \sum_{l_j=0}^{L_l-1} \sum_{u=2}^{N_u} \sum_{l=0}^{L_u-1} (\beta_{l,k})^1 \beta_{l,f,k} I_{l,f,k}^{u,1}$$

(12)

It can be shown that $E[I_{\text{MAI}}] = 0$. So, the variance of $I_{\text{MAI}}$ for a given channel realization is calculated as

$$\sigma_{\text{MAI}}^2 = E \left[ \sum_{k=0}^{K-1} \sum_{l_j=0}^{L_l-1} \sum_{u=2}^{N_u} \sum_{l=0}^{L_u-1} (\beta_{l,k})^1 \beta_{l,f,k} I_{l,f,k}^{u,1} \right]^2 = \sum_{k=0}^{K-1} \sum_{l_j=0}^{L_l-1} \sum_{u=2}^{N_u} \sum_{l=0}^{L_u-1} (\beta_{l,k})^1 \beta_{l,f,k} \left( I_{l,f,k}^{u,1} \right)^2$$

(13)

where

$$E \left[ \left( I_{l,f,k}^{u,1} \right)^2 \right] = E \left[ \left( \phi_1^2 + 2b_{l,k}^u \phi_1 \phi_2 + \phi_2^2 \right) (\tilde{R}_\Delta) \right]^2 + \left( \phi_1^2 + 2b_{l,k}^u \phi_1 \phi_2 + \phi_2^2 \right) (R_\Delta)^2 + 2R_\Delta \tilde{R}_\Delta \left[ \phi_1 \phi_2 + b_{l,k}^u b_{l,k}^u \phi_1 \phi_2 + b_{l,k}^u b_{l,k}^u \phi_1 \phi_2 + \tilde{R}_\Delta \phi_2 \right]$$

(14)

Due to the fact that $b_{l,k}^u$, $a_{l,k}^{u,1}$, and $\Delta$ are mutually independent, the evaluation of $E \left[ \left( I_{l,f,k}^{u,1} \right)^2 \right]$ can be done in three separately parts, as follows

$$E \left[ \left( I_{l,f,k}^{u,1} \right)^2 \right] = \left( a_{l,k}^{1,1} \right)^2 \left( R_\Delta \right)^2 + \left( \phi_1^2 + \phi_2^2 \right) (R_\Delta)^2 + \left( R_{\tilde{R}_\Delta} \right)^2$$

(15)

Hence

$$E \left[ \left( I_{l,f,k}^{u,1} \right)^2 \right] = \left( a_{l,k}^{1,1} \right)^2 \left( R_\Delta \right)^2 + \left( \phi_1^2 + \phi_2^2 \right) (R_\Delta)^2 + \left( R_{\tilde{R}_\Delta} \right)^2$$

(16)

Assuming random spreading sequences,

$$E \left[ \left( a_{l,k}^{1,1} \right)^2 \right] = \left( \sum_{j=0}^{N_l-1} a_{j,1} \right)^2 \left( \sum_{j=0}^{N_l-1} a_{j,2} \right)^2$$

$$E \left[ \left( a_{l,k}^{1,1} \right)^2 \right] = \left( \sum_{j=0}^{N_l-1} a_{j,1} \right)^2 \left( \sum_{j=0}^{N_l-1} a_{j,2} \right)^2$$

(17)

Similarly, it can be shown that

$$E \left[ \left( \phi_1 a_{l,k}^{1,1} \right)^2 \right] = N_s - \gamma$$

(18)

$$E \left[ \left( \phi_2 a_{l,k}^{1,1} \right)^2 \right] = \gamma - 1$$

(19)

$$E \left[ \left( \phi_2 a_{l,k}^{1,1} \right)^2 \right] = N_s - \gamma + 1$$

(20)

$$E \left[ \left( \phi_1 a_{l,k}^{1,1} \phi_2 a_{l,k}^{1,1} \right) \right] = 0$$

(21)

$$E \left[ \left( \phi_1 a_{l,k}^{1,1} \phi_2 a_{l,k}^{1,1} \right) \right] = 0$$

(22)
Desired user: \( l \), th path at the \( k \)th element

\[
\begin{aligned}
\mathbb{E}_{(a^1,a^u)} \left[ \mathbb{E}_{(b^1,b^u)} \left[ T_{l,f,k} \right]^2 \right] &= \left[ \gamma + N_s - \gamma \right] \left( \hat{R}_\Delta \right)^2 + \\
&\left[ \gamma - 1 + N_s - \gamma + 1 \right] \left( R_\Delta \right)^2 \\
&= N_s \left[ \left( \hat{R}_\Delta \right)^2 + \left( R_\Delta \right)^2 \right] \quad (23)
\end{aligned}
\]

Finally

\[
\mathbb{E} \left[ \left( T_{l,f,k} \right)^2 \right] = \mathbb{E}_{(a^1,a^u)} \left[ \mathbb{E}_{(b^1,b^u)} \left[ \left( T_{l,f,k} \right)^2 \right] \right] = \frac{N_s}{T_c} \int_{I_R} \left[ \left( \hat{R}_\Delta \right)^2 + \left( R_\Delta \right)^2 \right] dx \quad (24)
\]

Therefore

\[
\sigma^2_{MAI} = N_s \frac{T_c}{K-L_f-1} \sum_{k=0}^{K-1} \sum_{l_f=0}^{L_f-1} \left[ \beta_{u,k}^2 \beta_{1,k}^4 \right] \cdot I_R \quad (25)
\]

Similarly, the variance of the self-interference term has the following form

\[
\sigma^2_{SI} = N_s \frac{T_c}{K-L_f-1} \sum_{k=0}^{K-1} \sum_{l_f=0}^{L_f-1} \left[ \beta_{u,k}^2 \beta_{1,f}^4 \right] \cdot I_R \quad (26)
\]

These results in the equations (26) and (25) are the same obtained in [1], but in a different way, taking into account the statistical independence of \( b^i \), \( a^u \) and \( \Delta \). Due to the high number of multipath components and considering a relatively high number of users, the SI and the MAI can be modeled as Gaussian random variables. So, the probability of error conditioned to an instantaneous signal-to-noise ratio, \( \gamma_b \), is given by

\[
P_E | \gamma_b = Q \left( \frac{E_S}{\sigma^2_{total}} \right) = Q \left( \frac{E_S}{\sigma^2_S + \sigma^2_{MAI} + \sigma^2_{SI}} \right) \quad (27)
\]

Using the Monte Carlo method, the mean probability of error can be computed as

\[
P_E = \frac{1}{Z} \sum_{i=1}^{Z} P_E | \gamma_b^i \quad (28)
\]

where \( Z \) is the number of channel realizations.

VI. NUMERICAL RESULTS AND COMPARISONS

In this section, the Monte-Carlo simulation method was used in order to validate the previously derived semi-analytical results. The analysis in this paper is restricted to the CM3 channel model described in [5]. The number of multipath components was considered to represent 85% of the total channel energy. As can be seen in equations (25) and (26), the bit error rate (BER) performance depends on the \( I_R \) factor, that by its time depends on the shaping pulse continuous-time partial correlation. There are a significant number of pulse shapes used in impulsive UWB systems, such as: Gaussian pulse and its derivatives, Orthogonal modified Hermite pulses (MHP), damped sine pulses, etc [6]. One of the most useful pulse is the 2\textsuperscript{nd} derivative of the Gaussian Pulse, also known as Gaussian doublet, given by

\[
w(t) = \kappa \cdot \left[ 1 - 4\pi \left( t/\tau_p \right)^2 \right] \cdot e^{-2\pi \left( t/\tau_p \right)^2} \quad (29)
\]

where \( \tau_p \) is a time-scaling factor (similar to the pulse variance), and \( \kappa \) is a constant. Table II shows \( I_R \) for some useful pulse templates with normalized energy, pulse duration \( T_p = 2 n_s \), and \( \tau_p \) chosen to keep all the pulse with the same duration. The term \( I_R \) depends on the shaping pulse autocorrelation and figure 3 shows the normalized autocorrelation function for all pulses of table II.
As can be seen, increasing the order of derivative for Gaussian pulses or the order of MHP pulses, lower will be autocorrelation value, and consequently the $I_R$ value. Nevertheless, increasing these orders, the timing errors will increase [6], [7]. Additionally, to choose an ideal pulse shape it is also important to verify if it fits to the FCC spectral mask.

### TABLE II

$I_R$ FOR SOME PULSE SHAPES.

<table>
<thead>
<tr>
<th>Shaping Pulse</th>
<th>$\tau_p$ [ps]</th>
<th>$I_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Gaussian</td>
<td>288</td>
<td>$0.2099 \times 10^{-9}$</td>
</tr>
<tr>
<td>II. Gaussian 1st der.</td>
<td>288</td>
<td>$0.1494 \times 10^{-9}$</td>
</tr>
<tr>
<td>III. Gaussian 2nd der.</td>
<td>288</td>
<td>$0.1470 \times 10^{-9}$</td>
</tr>
<tr>
<td>IV. Gaussian 4th der.</td>
<td>288</td>
<td>$0.1448 \times 10^{-9}$</td>
</tr>
<tr>
<td>V. Damped Sine</td>
<td>312</td>
<td>$0.2458 \times 10^{-9}$</td>
</tr>
<tr>
<td>VI. MHP, order 1</td>
<td>57.9</td>
<td>$0.2028 \times 10^{-9}$</td>
</tr>
<tr>
<td>VII. MHP, order 2</td>
<td>57.9</td>
<td>$0.1508 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

The performance results were obtained considering random spreading sequences, $N_c = 63$, $N_a = 21$, $K = 1, 2$ and 3 antennas, $L_f = 5$ and 10, and $w(t)$ as a $2^{nd}$ derivative of the Gaussian Pulse with $T_p = 2\text{ns}$, $\tau_p = 0.2877\text{ns}$, which is a trade-off between correlation value and timing errors. The analytical results were obtained considering $\tau = 10,000$. Figures 4 and 5 show the performance results for 5 and 10 rake fingers, respectively.

From these performance results, one can see that the performance of a DS-UWB system can be improved by increasing the number of rake fingers and/or array elements. The good agreement between the analytical and simulation results shows that the Gaussian assumption for the MAI and SI is valid for the specific system configuration.

### VII. CONCLUSIONS

In this paper the performance of DS-UWB systems with temporal and spatial diversity was investigated. The results showed that the performance can be considerable improved increasing the number of rake fingers, but mainly the number of array elements. The results also demonstrated that the shaping pulse has influence on the system performance, and that, based on the correlation values and timing errors, the Gaussian $2^{nd}$ derivative pulse is a good choice. However, a better analysis for the shaping pulse considering correlation values, timing errors and fitting to the FCC spectral mask is needed. Additionally, one can see that the Gaussian assumption for the MAI and SI is a good approximation for this particular scenario.

### REFERENCES