Sets of Sequences for QS-CDMA Systems with Interference Cancellation over Multipath Rayleigh Fading Channels

André S. R. Kuramoto, Taufik Abrão, Paul Jean E. Jeszensky

Abstract—In this work Walsh-Hadamard, QS, Lin-Chang, LCZ-GMW, ZCZ sets of sequences are compared. The comparison is accomplished by analyzing the conventional receiver (Rake) and a parallel interference canceller (PIC) receiver performance using each one of these sequence sets in a multipath Rayleigh fading channel and similar system loads in quasi-synchronous condition.

Index Terms—DS/CDMA, quasi-synchronous, spreading sequence, parallel interference canceller.

I. INTRODUCTION

The DS/CDMA (direct sequence code-division multiple access) performance limitation is mainly determined by the multiple access interference (MAI), which is due to many users sharing the same bandwidth. This interference is the result of random delays among all active users' signals, making impossible the orthogonality maintenance among all spreading code waveforms. For multipath-fading channels the interference in the Rake detector's output signal is composed by MAI and self-interference (SI). The SI is composed by self inter-symbol interference (SII), due to multipath components from the previous or subsequent symbol, and self current-symbol interference (SCI), due to components from the current symbol. The multi-user detector (MuD) uses all the information from the other active users and also other estimates in order to cancel MAI and SII. The SCI can be beneficially used in the combining and decision stages. So a capacity increase can be obtained in comparison with conventional detection with a complexity increase expense. The parallel interference canceller (PIC) [1] [2] is a type of MuD where the interference from all other users are parallel and simultaneously estimated and subtracted. The first stage is a matched filter bank (MFB), as in the Rake detector, which generates estimates for all users' signals. In the next stage, the MAI and SII are reconstructed from all estimates obtained in the previous stage and then subtracted from the input signal, producing the next input signal with some residual error due to imperfect cancelling. This process can be repeated in multiple stages sending the desired user's signal with residual errors to a second MFB, cancelling stage and so on. The PIC with hard decision (PIC-HD) uses the sign(g) function in order to decide the estimated bit in all intermediate cancelling stages.

In this work we have considered a quasi-synchronous DS/CDMA (QS-CDMA) system. Due to the impossibility of perfect synchronism of all active users' received signals, the time delays among these signals will be considered as distributed, in an independent and uniform way, on the interval [0,τmax], where τmax represents the maximum synchronism error, inherent to the system. So, all used spread sequences should be almost synchronized and therefore the MAI and the SI can be substantially reduced by an appropriate choice of sequences.

Most of the related publications have just investigated the even correlation (EC) properties of sequences. However, for a complete analysis of the QS-CDMA systems we should consider not only EC properties but also the odd correlation (OC) properties of the sequences [3]. The OC functions affect the matched filter output when the information symbol changes inside the integration interval, while the EC functions affect the matched filter output when the information symbol doesn't change. Accepting that the information symbols are statistically i.i.d. (a reasonable hypothesis), the influence of OC is as important as EC in the MAI and SI determination and in the performance of the CDMA system. Therefore it is reasonable to investigate the EC properties so much as OC.

In this work QS-CDMA systems performance were compared with conventional detection (Rake) and MuD PIC-HD, in multipath Rayleigh fading channels, using the following sets of spreading sequences: Walsh-Hadamard, QS [4] [5], Lin-Chang [6], LCZ-GMW [7] and ZCZ [8] [9], with similar loading.

II. DEFINITIONS

The spreading sequences \( \mathbf{e}_i \) are defined as: \( \mathbf{e}_i = \{ e_{i,0}, e_{i,1}, \ldots, e_{i,N-1} \} \), where \( i \) represents the set's \( i \)-th sequence; \( N \) is the spreading sequence length; and \( e_{i,j} \in \{-1,1\} \) is the chip of the \( i \)-th sequence.

The ratio between the information symbol period, \( T_b \) and the chip period \( T_c \) is called processing gain \( G = \frac{T_b}{T_c} \). In this work, all chips of the sequence spread each one of information symbols, therefore \( G = N \).

The system loading \( Load = \frac{\#}{\#} \) relates the system active users number \# with the spreading sequence length \#N. The even cross-correlation (ECC) function can be defined as:

\[
R_{\hat{c}_i,j}(\tau) = \begin{cases} C_{\hat{c}_i,j}(N - \tau), & 0 \leq \tau < N \\ C_{\hat{c}_i,j}(\tau - N), & N < \tau < 0 
\end{cases}
\]

and the odd cross-correlation (OCC) function as:

\[
\hat{R}_{\hat{c}_i,j}(\tau) = \begin{cases} C_{\hat{c}_i,j}(N - \tau), & 0 \leq \tau < N \\ C_{\hat{c}_i,j}(-\tau), & N < \tau < 0 
\end{cases}
\]

where \( C_{\hat{c}_i,j}(\tau) \) is the aperiodic cross-correlation function, expressed by:

\[
C_{\hat{c}_i,j}(\tau) = \sum_{l=0}^{N-\tau-1} \sum_{l=0}^{N-\tau-1} H_{\hat{c}_i,j}(l)i(l)i(l+\tau) \quad 0 \leq \tau < N
\]

where \( i \neq j \); \( \tau \) means the delay between the spreading sequences, expressed in time units of chip, \( T_c \). When \( i = j \), we have the even and odd auto-correlation functions (EAC and OAC), (1) and (2), and the aperiodic auto-correlation function, (3), respectively.

III. SETS OF SPREADING SEQUENCES

A. Walsh-Hadamard Set of Sequences

Walsh-Hadamard sequences are derived from Hadamard codes [10]. The maximum system’s load with the Walsh-Hadamard (WH) set of sequences is \( Load_{max} = 1 \) because the sequences are obtained from lines (or columns) of the square matrix of Hadamard.

The ECC function for this set is zero when \( \tau = 0 \) because any two lines (or columns) of matrix of Hadamard are orthogonal. With this property a synchronous CDMA (S-CDMA) in a single path channel using WH sequences has virtually zero MAI. Nevertheless, when \( \tau_{max} \neq 0 \) or in a multipath environment, due to channel selectivity, the ECC and OCC functions can assume great values, implying a great MAI.

The EAC function for a WH sequence has peaks when \( \tau \neq 0 \). In this case the system allows a maximum synchronism error \( \tau_{max} \geq 1T_c \), problems will occur in the synchronism stage avoiding information recovery.

B. QS Set of Sequences

The QS set of sequences [4] [5] is composed by Gold sequences with suitable selected phases which results in a minimum ECC for small delays. In [4] it was shown that for Gold sequences the OCC distribution seems a Gaussian distribution with minimum variance when the ECC value also is minimum (\(-1\)). Therefore, for the Gold set of sequences in quasi-synchronism condition it is reasonable to...
adjust its phases in accordance with its ECC value. In [5], the quasi-orthogonality condition in a range $\tau$ (QQQS(r)) for QS sequences was defined resulting in $R_{i,j}(\tau) = -1$, for $\tau = 0, \pm 1, \ldots, \frac{2^n-1}{n}$.

The number of sequences in a set with properties QQQS(r) characterized with the length $N$ of sequences [5]. As the QS set of sequences is composed by Gold sequences, the EAC, with $\tau \neq 0$, and ECC possible values for a QS set with length $N = 2^n - 1$ with odd $n$ are: $-1$ and $\pm 2^{\frac{n+1}{2}} - 1$.

C. Lin-Chang Set of Sequences

The Lin-Chang set of sequences, proposed in [6], has balance and cross-correlation properties similar to a GMW subset of sequences constructed from the same primitive polynomial [11]. Lin-Chang sequences can be considered as a generalization of GMW sequences.

Given a primitive polynomial with grade $n$ and $K = \frac{q(n-1)}{2}$ seeds (or kernels) balanced sequences with length $2^m - 1$, with $m$ factor of $n$, we obtain a family of Lin-Chang sequences with length $N = 2^n - 1$. This family is bigger than the GMW subclass with similar cross-correlation properties with $2^{\frac{2(n-1)}{2}}$ distinct sequences, where $\phi(x)$ denotes the Euler function.

For $0 < |r| < \frac{2^n-1}{n}$ or $|r| \equiv 0 \mod \frac{2^n-1}{n}$, all ECC and EAC values of Lin-Chang sequences in the same family are minimum and equal to $-1$. However, in the same delay range $\tau$ the OCC values are not minimum.

There is compromise between the range $\tau$, where the ECC function value is $-1$, and the number of distinct sequences in the family [6]. Therefore, in order to obtain a maximum load with the Lin-Chang set of sequences we adopt $n = 2m$, reducing accordingly the delay range values where the ECC value is $-1$.

The ECC function may assumes a high value for $\tau = 0$, depending on the initial phase of the seeds. This means a high interuser interference when the users are in synchronism or near to this condition with delays restricted to a small fraction of the chip duration. Out of QS condition, when $|r| \geq \frac{2^n-1}{n}$, the ECC function for a Long-Chang set can also assume high values.

For $\tau \neq 0$ the EAC function of Lin-Chang sequences generated from $m$-sequences is reduced to $-1$, because in this case the generated sequence is a GMW sequence [6] [11]. When the sequences seeds are not $m$-sequences the Lin-Chang sequences auto-correlation shows other peaks with smaller magnitudes.

D. LCZ Set of Sequences based on GMW Sequences

For sequences in a LCZ (Low Correlation Zone) set based on GMW (LCZ-GMW) [7] sequences, the EAC and ECC functions are almost ideal for delay values in the range $|\tau| < LCZ$.

The EAC and ECC functions assume value $-1$ for $0 < |\tau| < T$ and $|\tau| < T$, respectively, where $T = \frac{2^n-1}{n}$, and $n$ and $m$ are integers which represent the degree of the primitive polynomials used in the construction of the GMW sequences that originate the LCZ-GMW set. The length of the LCZ-GMW sequences is given by $N = 2^m-1$.

In this work we just considered the case of $p = 2$ (binary sequences), resulting in $N = 2^m-1$ and $LCZ = T = \frac{2^n-1}{n}$.

According to [12], for a LCZ-GMW set of sequences with length $N$ there is a compromise between the size of the set $K$ and the LCZ value: $\frac{2^n-1}{n} \leq K$. Notice that for $n$ constant, the larger the value of $LCZ$, the smaller the value of $K$. Thus, the maximum delay for a LCZ sequences set with length $N$ is obtained when $n = 2m$, in which situation $LCZ$ is minimum.

E. ZCZ Set of Sequences

The EAC and ECC functions for the ZCZ sequences set are ideal for all delay values in the range $|\tau| < ZCZ$ [8].

In this work we considered the construction method III proposed in [8]. In this method, given $m$, $n$, and $t$, a ZCZ set composed of $K = 2^{n+m+t}$ sequences with length $N = 2^{n+m}+t+1$ and $ZCZ = \frac{2^{n+m}+t+1}{2}$ is obtained. In this way, systems that use this ZCZ set will have the maximum loading given by $Load_{max} = \frac{2^{n+m}+t+1}{2}$.

There is a compromise between the range where the EAC and ECC functions are ideal ($|\tau| < ZCZ$) and the number of sequences, with length $N$, in the set: $K ZCZ \leq N$ [12].

Table I shows the main characteristics of each previously described set of sequences.

IV. SYSTEM MODEL

The transmitted signal by the $k$-th user is given by:

$$s_k(t) = \sqrt{2P_k} \sum_{i} b_k^{(i)} a_k(t - iT_k) \cos(\omega_c t),$$

(4)

where $P_k$ represents the transmitted power; $b_k^{(i)}$ the $i$-th information symbol with period $T_k(\omega_c)$ is the carrier frequency; $a_k(t)$ corresponds to the spreading sequence defined in the interval $[0, T_k)$ and zero outside:

$$a_k(t) = \sum_{i=0}^{N-1} c_{k,i} p_{T_k}(t - iT_k)$$

(5)

where $c_{k,i} \in \{1, -1\}$ represents the $i$-th chip of sequence $k$ and $p_{T_k}(t)$ a rectangular shape pulse with unitary amplitude defined in the interval $[0, T_k)$ and zero outside. The processing gain, $\frac{2^n}{c_k}$ is equal to $N$.

The channel complex base band impulsive response in the instant $t_0$ is given by:

$$h_k(t, t_0) = \sum_{l=1}^{L} \alpha_{k,l} \delta(t - lT_k - \tau_{k,l,k})$$

(6)

where $L$ is the number of resolvable paths; $\alpha_{k,l} \cdot \tau_{k,l,k}$ and $\varphi_{k,l,k}$ represent the channel’s coefficient, delay and phase, respectively, for the $l$-th component of the $k$-th set; it is assumed that phases $\varphi_{k,l,k}$ are uniformly distributed in the interval $(-\pi, \pi)$; $\tau_{k,l,k} = \tau_k + \Delta_l$, where $\Delta_l$ are uniformly distributed in the interval $[0, \tau_{max}]$ and $\Delta_l$ is the $l$-th multipath component delay, given a specific power-delay profile.

Considering that the small scale-fading envelope follows a Rayleigh distribution, the channel coefficient’s amplitude, $\alpha_{k,l}$, probability density function (PDF) is given by: $f(x) = \frac{2x}{\alpha^2} e^{-\frac{x^2}{\alpha^2}}$, where $\alpha$ is the coefficient’s module and $\rho$ the multipath’s component average power $\rho = E[\alpha^2]$.

The base band signal that reaches the receiver can be written as:

$$r(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{i} \left[ \sqrt{2P_k} \alpha_{k,l} b_k^{(i)} \right] \delta(t - lT_k - \tau_{k,l,k}) + n(t)$$

(7)

where $n(t)$ is the channel’s Additive White Gaussian Noise (AWGN) with bilateral power density given by $\frac{N_0}{2}$ and $\varphi_{k,l,k} = \varphi_{k,l,k} - \omega_c \tau_{k,l,k}$.

Considering a coherent reception the $t$-th matched filter output, matched to the $t$-th received bit information from the $k$-th user’s multipath component (finger), is given by the terms:

$$z_{k,l,k}^{(i)}(t) = \sqrt{\frac{P_k}{2}} T_k \alpha_{k,l} b_k^{(i)} + SI_{k,l}^{(i)} + I_{k,l}^{(i)} + N_{k,l}^{(i)}$$

(8)

where the first term corresponds to the desired signal, the second to the auto-interference, the third to the MAI over the $l$-th multipath component of $k$-th user and the last to the filtered AWGN.

The auto-interference term, $SI_{k,l}^{(i)}$, can be written as:

$$SI_{k,l}^{(i)} = \left\{ \begin{array}{ll}
\sqrt{\frac{P_k}{2}} T_k \alpha_{k,l} b_k^{(i)} & R_{k,k,h}(\tau_{k,l,k} - \tau_{k,l,k} - 0)
+ b_k^{(i)} R_{k,k,h}(\tau_{k,l,k} - \tau_{k,l,k} - 0)
+ b_k^{(i)} R_{k,k,h}(\tau_{k,l,k} - \tau_{k,l,k} - 0)
+ e^{-i\phi_{k,l,k}} \tau_{k,l,k} \leq \tau_{k,l,k};
\end{array} \right.$$
TABLE I

| Set of sequences | $\text{Load}_{\text{max}}$ | $\max |R_{i,j}^{(\tau)}|$ | $\min |R_{i,j}^{(\tau)}|$ | $\tau$ | $\min |R_{i,j}^{(\tau)}|$ | $\tau$ | $\max |R_{i,j}^{(\tau)}|$ |
|------------------|------------------|------------------|------------------|------|------------------|------|------------------|
| WH | $N$ | $1$ | $2^{\frac{m+1}{2m}} - 1$ | $1$ | $|\tau| \in [0;1]$ | $|\tau| = 0$ | $|\tau| = 0$ |
| QS,QOQS(5) | $\frac{2^{m+1}}{2m}$ | $1$ | $|\tau| \in [1; 2^{m+1} - 1]$ | $|\tau| = 0$ | $|\tau| = 0$ | $|\tau| = 0$ |
| Lin-Chang | $\frac{N+1}{L_{\text{ex}}}$ | $-1$ | $|\tau| \in [0;1]$ | $|\tau| = 0$ | $|\tau| = 0$ | $|\tau| = 0$ |
| LCZ-GMW | $\frac{N+1}{L_{\text{ex}}}$ | $0$ | $|\tau| \in [0; 2^{m+1} - 1]$ | $|\tau| = 0$ | $|\tau| = 0$ | $|\tau| = 0$ |
| ZCZ | $\frac{N+1}{L_{\text{ex}}}$ | $1$ | $|\tau| \in [0; 2^{m+1} - 1]$ | $|\tau| = 0$ | $|\tau| = 0$ | $|\tau| = 0$ |

Finally, the Maximal Ratio Combiner (MRC) combines the $D$ correlators' output signals, $\hat{y}_k^{(\tau)}(s) = \sum_{\tau} \text{Re} \left\{ \hat{y}_k^{(\tau)}(s) \hat{y}_k^{(\tau)} \right\}$, followed by an abrupt decision circuit, $\hat{y}_k^{(s)}(s) = \text{sign} \left( \hat{y}_k^{(\tau)}(s) \right)$.

V. NUMERICAL RESULTS

In order that the performance comparison be fair among the various QS-CDMA systems, the sets of sequences in all Monte-Carlo simulations were chosen in a mode that assures us of similar loading. For Lin-Chang set we adopted $m = 3$, $n = 2m = 6$. The primitive polynomial $x^9 + x^6 + x^3 + x + 1$ was used for the GF($2^9$) field construction. For performance determination in each iteration we randomly selected four sequences from the five available sequences.

For the LCZ-GMW set we adopted $p = 2$, $n = 6$, $m = 3$ and the same primitive polynomial as before.

For the ZCZ set we adopted $m = 4$, $n = 1$ and $t = 1$, resulting in a set of 4 sequences with length $N = 64$ and $L_{\text{ex}} = 9$. The ECC function for this set will be minimum, 0, for $|\tau| < 8$.

The QS set was derived from the Gold(203,277) set. From this Gold set 4 subsets are obtained with 8 sequences each with length $N = 127$ and property $QOQS(5)$. Arbitrarily we chose the subset $Q_1$, that given all 4 subsets present similar correlation properties.

For WH set we adopted $N = 64$ and in the performance determination 4 sequences were selected from the available set in each iteration.

Table II synthesizes the main parameters for the selected set of sequences: processing gain $N$, the system loading, the number $U$ of active users in the system, the maximum values for $R_{i,j}^{(\tau)}$ and $\tilde{R}_{i,j}^{(\tau)}$ considering $0 \leq \tau < N$ and the interval where ECC is maintained minimum.

Table III shows the power-delay profile that was adopted for the performance analysis in a multipath-fading Rayleigh channel. This typical urban profile was based on the COST207 study and has a reduced number of multipaths in order to alleviate simulation's complexity and processing time.

In all simulations we considered a perfect power control scenario ($P_1 = P_2 = \ldots = P_U = P$). The phases, amplitudes, delays and channel coefficients were assumed as exactly known. Other adopted parameters were: carrier frequency of $f_c = 2 \, \text{GHz}$, mobile velocity of $v = 110 \, \text{km/h}$, resulting in a maximum Doppler frequency of $f_m = \frac{v}{c} = 203.5 \, \text{Hz}$, and Rake diversity of $D = 4$, because with 4 fingers it is possible to capture more than 90% of the total energy.

The MAI term, $I_{k,h}^{(\tau)}$, can be written as:

$$I_{k,h}^{(\tau)} = \sum_{\nu \neq k} \sum_{L=1}^{D} \left\{ \sqrt{P_2} \left[ \hat{y}_k^{(\tau)}(s-1) - \hat{y}_\nu^{(\tau)(s-1)} \right] \right\}$$

where $D$ represents the number of correlators in the receiver for each user, also named Rake diversity, which in a real system needs the estimation of the following parameters for all users: channel coefficients, $\hat{\sigma}$, power, $P$, delay, $\tau$, (and therefore correlations, $\hat{R}$), phase, $\phi$, and the previous cancelling stage information bits, $\hat{u}(s-1)$.

The MAI estimate obtained in the $s$-th cancelling stage, $\tilde{I}_{k,h}^{(\tau)}(s)$, can be written as:

$$\tilde{I}_{k,h}^{(\tau)}(s) = \sum_{\nu \neq k} \sum_{L=1}^{D} \left\{ \sqrt{P_2} \left[ \hat{y}_k^{(\tau)}(s-1) - \hat{y}_\nu^{(\tau)(s-1)} \right] \right\}$$

The PIC $s$-th stage output signal, considering the $t$-th multipath component for $i$-th information bit of the $k$-th user results:

$$\text{Z}_{k,h}^{(\tau)}(s) = \text{Z}_{k,h}^{(\tau)}(0) - \tilde{I}_{k,h}^{(\tau)}(s) - \hat{I}_{k,h}^{(\tau)}(s)$$

The considered multistage PIC-HD removes the interference from the estimated self-interference interference (SIH) and MAI in $s$ stages. In the first stage, $s = 1$, the estimates are obtained from the correlators’ outputs, stage $s = 0$. The SIH estimate, obtained in the $s$-th cancelling stage can be written as:

$$\tilde{S}H_{k,h}(s) = \sum_{L=1}^{D} \left\{ \sqrt{P_2} \left[ \hat{z}_k^{(\tau)(s-1)}(s-1) \right] \right\}$$

Finally, the Maximal Ratio Combiner (MRC) combines the $D$ correlators’ output signals, $\hat{y}_k^{(\tau)(s)}(s) = \sum_{\tau} \text{Re} \left\{ \hat{y}_k^{(\tau)(s)}(s) \hat{y}_k^{(\tau)(s)} \right\}$, followed by an abrupt decision circuit, $\hat{y}_k^{(\tau)(s)}(s) = \text{sign} \left( \hat{y}_k^{(\tau)(s)}(s) \right)$.

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In set of sequences constructed with the primitive polynomials $x^7 + x^3 + x^2 + x^2 + x + 1$, or 203 and 277 in octal notation, respectively.
Fig. 1. $\frac{BER \times E_b}{N_0}$ performance for the Rake receiver with MRC and the Rake receiver with multistage PIC-HD, using the ZCZ set of sequences; $\tau_{\text{max}} = 2T_c$.

Performance results were obtained in terms of average bit error rate ($BER$). Figures 1, 2, 3, 4 and 5 show the $BER \times \frac{E_b}{N_0}$, where $E_b = P \cdot T_b$, performance results obtained via Monte-Carlo simulations. For sequences with length $N = 63$ and $N = 64$, it was considered that $\tau_{\text{max}} = 2T_c$ and for sequences with length $N = 127$, it was considered that $\tau_{\text{max}} = 4T_c$, resulting in an equal maximum relative delay in all simulations. The maximum relative delay is defined as a function of sequence length as: $\tau_{\text{max}} = 2^{-\ell} \times 100 \%$, and allows us to compare the asynchronism effect on systems with different spreading sequence lengths ($N$). For comparison purposes the analytical results for a single user case, considering a Rake receiver with MRC and diversity $D$, was included (single user bound, SUB)[10]: $BER_{\text{SUB}} = \frac{1}{2} \sum_{D}^{D} \left[ 1 - \sqrt{\frac{1}{\pi^2 D^2}} \right] \prod_{i=1}^{D} \frac{\tau_{\text{max}}}{\tau_{\text{c}}}$.

In AWGN and multipath-fading channels the QS-CDMA system with PIC-HD detection presents better performance than the conventional detection (single matched filter and matched filters followed by MRC, respectively).

For QS-CDMA systems with Rake detection and MRC the best performance is obtained with the ZCZ set of sequences, figure 1, followed by the LCZ-GMW set, figure 2, and the QS set, figure 3. Now for the Lin-Chang set of sequences, figure 4, the performance is unsatisfactory and close to that obtained with the WH set, figure 5.

Note that the PIC-HD detector performance with Lin-Chang and QS sets of sequences is similar. These results show that the major complexity associated to the PIC-HD detection, operating in a multipath-fading channel, reduce or even eliminate small performance differences observed in relation to the Rake receiver with MRC, for these two sets of sequences. Finally the WH set performance is unsatisfactory even with PIC-HD detection.

Due to low system loading (imposed by the LCZ-GMW set) only one stage in the PIC-HD is sufficient in order to obtain a great improvement in comparison with Rake with MRC. With this low load condition it is observed that no additional gain is obtained with more stages in the PIC-HD.

Figure 7 shows the average performance as a function of users asynchronism level, in a Rake receiver with MRC, considering all five sets of sequences with similar loading, table II. The ZCZ set has presented the best relative performance. Almost for all intervals, its performance is very close to the $SUB(D = 4)$ bound, showing relative system robustness against asynchronism errors (at least 16%).

TABLE III

<table>
<thead>
<tr>
<th>$\ell$</th>
<th>$\Delta x_i$</th>
<th>$E_{\text{av}}[\alpha_i]$</th>
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<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>0.185</td>
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<tr>
<td>2</td>
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<td>0.379</td>
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<tr>
<td>6</td>
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<td>0.337</td>
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even in a channel with many multipaths. Performance degradation using a Rake receiver with MRC is observed with the LCZ-GMW and QS sets of sequences, not only confronting with the ZCZ set but also increasing the synchronization error. The WH set has the worst relative performance maintained practically constant for all synchronization error range.

In contrast to other sets behaviour the Lin-Chang set shows a growing average performance with the \( \tau_{\text{max}} \) increase tending to reach the QS performance. This behaviour can be explained through the non-optimal cross-correlation characteristic for small delays (\( |\tau| < 1 \)) [6].

In contrast with the results obtained for the WH set in a single path channel, figure 7 shows a non-optimal performance even for \( \tau_{\text{max}} = 0 \) which means perfect synchronism condition. This performance can be explained by observing that due to the multipath propagation the orthogonality condition among components are lost.

A similar problem occurs using the QS set of sequences. For instance, the good minimum ECC characteristics when \( |\tau| \leq 2T_c \), for the set with property QOQS(5), used in simulations, is not sufficient in a multipath-fading channel due to various multipath components with high delays.

Finally, figure 6 presents the performance results for the five sets of sequences associated with PIC-HD with 1 cancelling stage as a function of the relative synchronism error. It is observed that for the same Rake diversity, \( D = 4 \), the performance differences with MuD are minimized and, consequently, the correspondents BER result closer to the SUB bound with diversity \( D = 4 \). It is noted that even increasing the relative synchronism error there is no degradation in the observed performance.

VI. CONCLUSIONS

The main characteristics of five sets of sequences, recently proposed in the literature for QS-CDMA applications, were analysed and compared.

The main correlation properties for these sets were investigated in a quasi-synchronism scenario. Performance results for QS-CDMA systems with conventional and multi-user detection in a multipath-fading channel were obtained, via Monte-Carlo simulation, in order to compare these sets of sequences.

The ZCZ set of sequences with conventional detection, and also with MuD, has the better relative performance.

With a suitable choice for the set of sequences only one stage is sufficient for QS-CDMA systems with PIC-HD in order to obtain a significant performance improvement in relation to the Rake receiver with MRC. This improvement means a small increase in the receiver complexity making it viable for implementation in the system’s base station side.

REFERENCES


